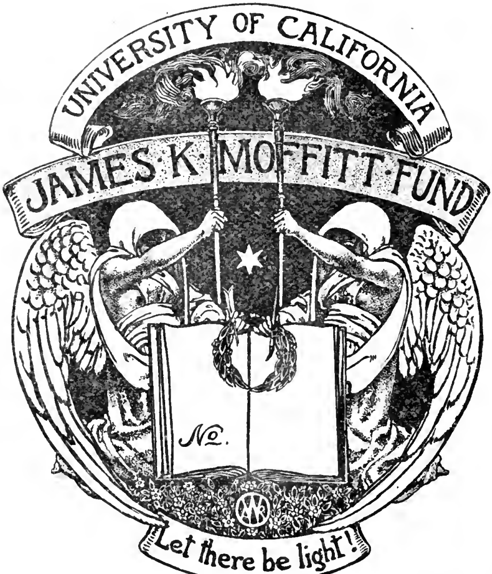
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LOGIC.

LONDON: GEORGE BELL AND SONS
PORTUGAL ST. LINCOLN'S INN, W.C.
CAMBRIDGE: DEIGHTON, BELL & CO.
NEW YORK: THE MACMILLAN CO.
BOMBAY: A. H. WHEELER & CO.

LOGIC

AN INTRODUCTORY MANUAL FOR THE USE OF UNIVERSITY STUDENTS

BY

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LONDON

GEORGE BELL AND SONS

1908

First Published, 1896.
Reprinted, with corrections, 1900, 1908.

PREFACE.

THE present work is intended to serve as an introductory manual for the use of students.

Like the author's "Psychology" and his "Ethics" it follows, in the main, the customary and traditional lines adopted by most English teachers, and authoritatively laid down by the English universities. In particular the author believes the book will prove of service to those reading Logic for the London B.A. Pass Examination.

At the same time attention has been called to certain aspects of the subject which seem of great importance, but which have apparently escaped the notice of examiners. Thus in Chapter VII. there is a brief exposition of the relation between the purely formal Laws of Thought and the regulative principles that underlie actual thinking about concrete things, in which the conditions of abstract thought are never entirely fulfilled.

Obligations to previously published works are fully acknowledged in the text; but the author desires here to express his thanks to Mr. W. E. Tanner, M.A., of the University of London, who has very kindly assisted him with many useful corrections and suggestions.

PUTNEY,
January, 1896.

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LOGIC.

CHAPTER I.

THE SCOPE OF LOGIC.

§ 1. Inference.

When from any piece of knowledge a fresh piece of knowledge is arrived at without our having any new experience, we have inference. From experiments made with the finger and the top bar of the grate, a child comes to the knowledge that if at any future time he puts his finger on the bar when the fire is alight he will get burnt—this is inference. The inference may be wider: he may arrive at the knowledge that if he puts his finger on *any* bar of *any* grate, when the fire is alight, he will get burnt. But in either case he gets at fresh knowledge without any further appeal to experience.

This process takes many different shapes In its simplest form it is practically unconscious; it is, at any rate, performed with the minimum of consciousness. Much of what passes as direct perception turns out to be really inferential. Psychology shows us that

when we perceive an object, the mind almost unconsciously goes through a complicated process which is, in many respects, analogous to fully conscious inference. On a small basis of sensation we instantly build up a big superstructure of perception. We see a minute patch of greyish-blue colour on the horizon, and instantly say, "I see a church." This practically involves a large number of inferences. We unconsciously jump to the conclusion that if we came up to the greyish-blue patch, we should find a large stone building of a particular kind. There has been unconscious, or as it is better called, sub-conscious, inference in localizing the cause of the impression, and in recognizing the object as one of a class. Not even the simplest act of perception is free from the element of inference; and the mistakes which occasionally occur are evidence of this. When I recognize, or think I recognize, my friend Jones across the street, I proceed from a piece of actual experience—certain visual sensations—but I erect on this a mass of inferences which may be erroneous. The knowledge I get is out of all proportion to the foundation of actual experience on which it rests.

There is a great deal of inference of a somewhat more conscious kind, which, however, is still so simple as to be scarcely recognized as inference. Such elementary acts of thought are seldom translated into words, but are registered, as it were, automatically by our mental machinery. In this way the child and the dog lay up for themselves a great deal of knowledge

from a small basis of experience. Even the application of old knowledge to a new case involves inference of this sort. A great deal of what is called "intuition" is inference of this kind. The rapidly-formed judgments of the clever business man, or the artist, or the great military commander, are frequently of this nature. They seem almost to be direct perceptions; the grounds on which they rest, and the steps by which they are reached, cannot be distinguished by those who form them. Only the skilled psychologist can hope to detect them.

Finally, there are the fully conscious inferences which are ordinarily expressed in words. They vary enormously in complexity. Some are quite simple and concrete, and scarcely need the use of formal language at all: others are extremely complex or extremely abstract, and involve the employment of symbols more exact than those of everyday language—*e.g.*, the symbols of mathematics. It is with these fully conscious inferences that Logic has to do.

§ 2. The Validity of Inference.

As we have seen, we are constantly making inferences. Many of these inferences are so rapid and so little conscious that it seems impossible to criticise them. Others are made slowly and with difficulty in the full light of consciousness; they can be easily reproduced and examined.

Logic is the science which lays down the general

conditions of valid, conscious inference. It undertakes to explain on what condition conscious inferences may be rightly drawn.

By valid inference we mean inference which gives us results in accordance with experience as a whole; not my experience only, but that of all men. If from the proposition, "All fires burn," I draw the conclusion that this fire will *not* burn, I obviously draw a conclusion which my own experience and that of others will easily show to be false. If from a consideration of a triangle I come to the conclusion that all the angles are together equal to one right angle, I shall have a result which I myself shall probably abandon on further reflection, and of whose truth I shall at any rate never succeed in convincing any qualified person.

The final appeal of the validity of an inference must be to the thought of all competent persons. What the experts, after careful examination, declare to be valid, that must be accepted as valid; and beyond the judgment of experts—which includes every person capable of thinking and reflecting on his thought—we cannot go. If the question is raised, "Who is to decide whether a given person who disagrees with the rest of mankind on a given inference, is 'capable' or not?" we can only reply once more, the consensus of experts. The inference is right because the qualified judges accept it; and the judges can only be admitted as qualified by their agreement with the normal standard of judgment.

This, after all, is our last resort, not only in matters of truth, but in matters of taste and conduct. Art and Morality, as well as Science must appeal to the old test of Catholic orthodoxy, *quod semper, quod ubique, quod ab omnibus.*

Fortunately the number of abnormal persons in the world is less in the case of knowledge than in the case of art and morality. Where inclination and interest do not distort the judgment, most persons (not obviously insane or idiotic) will arrive at the same conclusion, provided the question be sufficiently simple to admit of a full knowledge of its conditions. From these results we can generalize and lay down canons by which we shall be able to test all inferences in which the conditions are similarly simple. The minority which really disputes the multiplication table, or the rules of deductive inference, is microscopic. The conditions of quantitative inference laid down by the mathematician, and the still more general conditions of qualitative inference laid down by the logician, have practically never been challenged; though some would-be reformers have proposed to express them in a somewhat different way, and others have been unable to see the advantage of laying them down at all.

Most of the inferences we make, even on comparatively difficult subjects, are made very rapidly, and though we can often bring into full consciousness the steps which led to our conclusion, we do not necessarily do this. If, however, the conclusion at which we arrive dissatisfies us, if we feel inclined to challenge

it, we re-state the grounds to ourselves and examine them. As thus re-stated, the grounds are necessarily not quite the same as before. Considerations which did not come into full consciousness are now brought clearly forward in a definite shape. They must be put in such a way as to make themselves obvious to anybody and everybody who cares to inquire into the matter. Logic, then, does not profess to represent the actual process by which conclusions are arrived at, but the process by which they must be arrived at, if they are to be accepted by everybody. I may know, in an indefinite and confused fashion, certain things about a person accused of a crime, and this knowledge may make me feel certain that he is guilty of that crime. It is not, however, until the facts are marshalled in a certain way, and their dependence on other facts clearly shown that I can consider the case *proved;* because it is not till this has been done that all other persons will necessarily draw the same inference from them that I immediately did.

When the grounds on which a conclusion rests have been displayed in such a way that the vast majority of sane persons will inevitably draw that conclusion, we say that the conclusion is *proved.* An inference may be valid and yet not be proved ; but if it is valid it must be susceptible of proof. Unless we can believe it possible to give grounds for accepting it which shall be objectively valid—*i.e.*, valid for everybody, at any rate all experts—we cannot regard it as a valid inference.

We may now amend our former definition, and say that Logic is the science which deals with the conditions of *proof*, instead of the conditions of valid inference.

§ 3. Material and Formal Inferences.

If some one tells me that he has a collie, I can at once infer that the animal will have certain attributes—say a head and tail, four legs, long silky hair, and so forth. This inference depends on my knowledge of collies. I have seen a good many, I have seen pictures of a good many, and heard about a good many, and I have read descriptions of them in books. These grounds for my new, inferred knowledge I can put before myself in words; but they finally rest on experience. By the use of my senses I have acquired them, either directly or indirectly; by my own observation of collies, or through the observation of friends, artists, and authors. This kind of inference arising from an acquaintance with the subject matter, or content, of our objects of thought is called *material inference*. From knowledge of other collies I get to have knowledge of this new collie which I have never seen; a new case is reached.

If, however, I have come to know that "No collies are crustaceans," I can at once infer that my friend's collie is not a crustacean, even if I do not clearly understand what a collie is, or what a crustacean is. Here my inference depends, not on my knowledge

of collies, but on the form of the proposition as presented to me. The certainty of the inference is not diminished by my ignorance. It is independent of the matter, or content, of the thought. It can be expressed symbolically. If "No X is Y," then "this X is not Y." The conditions of these *formal inferences* can be laid down quite clearly and definitely, and with much more detail than those of material inferences, and therefore the greater part of most manuals of Logic is taken up by them.

But the material inferences are at least equally important. The amount of fresh knowledge to be got by manipulating the *form* is after all very limited; and fresh material of knowledge has constantly to be got by experience. Yet that of which we have direct experience, that is, immediate perception by our own senses, is necessarily very small. To settle under what conditions we may extend our limited experience to fresh cases, though a difficult task and impossible of entire fulfilment, is so important that it cannot be set aside.

Some logicians, *e.g.*, Mill, have refused the name *inference* to formal inferences, on the ground that no really new knowledge is obtained in them. But by new knowledge they mean knowledge of new cases, not previously included. If from the truth that "All collies are silky-haired" I arrive at the truth that my friend's collie is silky-haired, I do not get at a fresh case, because my friend's collie is included in the *all.* And if I infer from it that "*Some* silky-haired animals are collies," I again do not get beyond the original set of

cases included in the original statement. Such inferences Mill regards as unworthy of the name; they are "mere verbal transformations."

If, however, we are going to refuse the name "inference" to these cases, we shall have to go a great deal further. When a mathematician solves an equation he gets at no fresh fact. However complex the original equation, he gets at the meaning of x (which was there all the time) by eliminating what he does not want, until he gets x on one side of the equation and its value on the other. It would be an absurd abuse of language to declare that the process of solving a difficult quadratic equation involves no inference, because the value of x was all along implied, and we have simply disentangled this from its accompaniment.

When from a given judgment or judgments we arrive at another judgment, differing from the given judgments in form or in matter, we have inference. In formal inference, as well as in material inference, there is a real progress of thought from what is known to what is unknown. Omniscience might correctly discern all the formal implications of a proposition, as it might do the value of x in an equation, in one and the same act of thought; but men have to pass with some difficulty from a statement to the statements which are implied by its form. That the process is not always easy will be seen when we come to Chapters VIII. and IX.

§ 4. Deduction and Induction.

Deduction is formal inference from a proposition to one as general or less general. If I pass from the statement that "No dogs are crustaceans" to the statement that "No crustaceans are dogs," I pass to one of the same degree of generality as the original. I refer in each case to the whole of the dogs and the whole of the crustaceans, and exclude them from each other. If I pass from "All men are mortal" to "Socrates is a mortal," I pass from a general truth to a particular case. Both inferences are formal in character ; and no fresh cases are involved.

When from observing many dogs I come to the conclusion that "*all* dogs are mammals," I exceed the limits of my knowledge by going to fresh cases of which I know nothing. My new proposition extends much farther than I or anyone else can ever profess to have direct knowledge. The conclusion is wider than the knowledge on which it is based. The process by which we arrive at it is called *Induction.*

Under the head of Deduction we include most formal inference. We may deductively infer a new conclusion :

(1) From a single given proposition ;

(2) From two or more given propositions.

The first is called *immediate inference;* the second *mediate inference.* Some writers prefer to place *immediate inference* in a separate class, apart from deduction as well as induction. Mr. Welton calls them (or some of them) *eductions.*

Most inductions are material; but one particular type of induction belongs to the class of formal inferences. (See Chapter XII. § 2.)

§ 5. Logic as a Science and an Art.

Logic teaches us the conditions of *proof*. It analyses our thoughts from this point of view. It does not profess to exhibit what consciously or unconsciously takes place in our mind when we think. That task belongs to Psychology. Logic shows under what conditions one may draw valid inferences.

Logic, as it gives an account of the conditions of valid inference, is a science. A Science is a systematic account of facts of any kind, with an explanation of those facts whenever explanation is possible. The arrangement of facts which it adopts is dictated only by speculative interest. It employs definition, and classification, and reasoning only in order that the facts may be understood.

An Art keeps practice always in view. It classifies, and defines, and reasons, with a view to arriving at *rules* and not at general truths.

Many studies partake of the nature both of science and art. An ordinary text-book on Arithmetic or on Music gives rules as well as principles. In such a book we find discussion of general principles of number or of harmony; but it devotes more attention to rules for working exercises, and to attaining facility in certain operations. In an ordinary treatise on Logic more

space is devoted to the scientific treatment of the conditions of proof, and less to the practical rules for avoiding error in reasoning. But both aspects of the matter are considered.

All arts are based on science, and in the case of Logic the scientific basis is very prominent and very important. If error in reasoning were impossible, a science of Logic might still exist, though it would cease to have any practical value; but there would be no art of Logic to teach us to do properly what we could not avoid doing properly.

As an art Logic is useful, for it shows us the likely sources of error, and tells us how to avoid them. It gives us methods of testing reasonings of whose validity we feel any doubt. It exercises our intellects in the way best calculated to strengthen them, and thus furnishes a course of mental gymnastics devised on scientific principles. One man may of course be stronger without bodily gymnastics than another man with their assistance. But gymnastics are of service in strengthening the weak, and even adding to the strength of the strong. If good, they develop all the muscles of the body, and do not permit some of them to degenerate because they are not required in everyday use. Just so it is with Logic. Some who have studied Logic are bad reasoners; and some men who have never opened a Logic book are good reasoners. But Logic as an art improves all in some degree; and it insures that the reasoning powers shall be exercised in all sorts of ways. The tendency of particular

studies is often to develop ability along certain special lines at the expense of ability along other lines. Most scientific men are accurate thinkers within their own sphere; but their training does not always enable them to reason fairly and clearly about other subjects. The metaphysician sometimes makes a bad biologist; and the biologist is not always a conspicuous success as a metaphysician. The lawyer, when he applies his mind to politics, is often inclined to demand a kind of proof of which the subject is incapable.

In fact, one of the services which Logic performs is to show us the limitations of knowledge, and especially to make clear to us the *sort* of proof of which any given subject is capable. The mathematician, the biologist, and the politician deal with subjects which are susceptible of proofs of very different kinds.

§ 6. Logic and other Sciences.

As a pure science Logic deals only with the general conditions of proof. It has nothing to do with the particular conditions of special inquiries. It is even more abstract than mathematics, which deal with the relations of quantity and number. Logic deals only with the relations of co-existence and likeness. Mr. Spencer says that Logic "concerns itself with the most general laws of correlation among existences considered as objective." This, however, is not quite true, since it is concerned only with these in so far as they afford ground for the criticism of inferences.

Logic is marked off from Metaphysics by the fact that while Metaphysics is a criticism of knowledge as a whole, Logic deals only with the process of inference, and those processes which are subsidiary to inference, such as definition and classification. Metaphysics takes into account the ultimate basis of our knowledge. It examines and tries to explain the meaning of those ideas which underlie all our knowledge, the practical knowledge of the business man as well as the speculative knowledge of the savant. It analyses such concepts as Identity, Time, Space, Substance, Cause, Subject, and Object, many of which play no part in Logic. The general conditions of inference are independent of Time and Space; and although Identity, Substance, and Cause need to be examined by the logician more than by the man of science or the practical man, his analysis of them does not go very deep. He is only concerned to find a working account of them, such as may subserve his purpose; their full analysis does not trouble him.

The relation of Logic to Psychology has been several times touched on. The logician has nothing to do with large portions of the territory of mental science. Will and Emotion, and even Perception and Memory, concern him very little. Even with regard to Reasoning he does not try, as the psychologist does, to represent what actually goes on in the mind. He does not classify and explain mental phenomena as such; but he lays down the form which certain mental activities must be made to take if they are to be

regarded as valid. Psychology is descriptive and explanatory ; Logic is regulative.

Logic deals with thoughts expressed in language or some symbolical equivalent of language. The mere concept, or judgment, or reasoning as it exists in the mind is not the subject matter of the logician; but the concept, judgment, or reasoning expressed in words, or some substitute for words.

CHAPTER II.

TERMS.

§ 1. Propositions.

A JUDGMENT expressed in words is called a proposition. The logician analyses a simple or categorical proposition into three parts: Subject, predicate and copula.

Snow . . .	is . . .	white
(subject)	(copula)	(predicate)
Snow . . .	is not . . .	hot
(subject)	(copula)	(predicate)

The subject is the name of that about which the assertion is made; the predicate is the expression of that which is asserted about the subject; the copula (a part of the verb *to be*) is the mere symbol of assertion.

Confining ourselves to simple propositions, we may say that only propositions of the type *S is P* (or *S is not P*) are in strictly logical form. Here the predicate is quite detached from the copula; whereas in ordinary language the two are blended and confused by the use of verbs which not only show that an assertion is made, but assert something specific. Thus

the logician and the grammarian differ in their analysis of a proposition. The latter is concerned with the actual usages of language and is not concerned with the possibility of drawing correct inferences, so he does not trouble to separate the copula *is* or *is not* from the predicate. The former does not mind his analysis appearing uncolloquial and uncouth ; but he is extremely anxious to see exactly *what* is predicated. Let the proposition to be analysed be :

"The curfew tolls the knell of parting day."

The grammarian calls everything after the word *curfew* the predicate, and subdivides this up into verb and object. The logician arranges it thus :

"The curfew—is—tolling the knell of parting day,"

and calls all after the word *is* the predicate.

§ 2. Terms.

The subject and the predicate are called the terms of the proposition, because they form the ends or *termini* of it. In practice, however, *term* is used for any word, or group of words, used as a subject or predicate of a proposition, or capable of being so used. Every term then is a name, which can be made the subject of an assertion or the predicate of an assertion It may consist of one word or many. *Victoria*, and *the present Queen of England and Empress of India* are both single terms, though one consists of only one word

and the other of nine ; both refer to and denote only one and the same object.

A word which when standing alone can be used as a term (subject or predicate) is called a *categorematic* word (Greek κατηγορέω, I assert). A word which requires the addition of a categorematic word to enable it to form a term is called a *syncategorematic* word (Greek σύν, with, and κατηγορέω). All terms, then, consist of categorematic words, such as nouns (substantive or adjective), pronouns and participles, either in whole or in part. Adverbs, prepositions, conjunctions, and interjections, and also purely demonstrative adjectives are syncategorematic. They cannot be used as the subject or predicate of a proposition as long as they are used in their proper sense. It is true we may say "*Of* is a preposition," but here *of* is not used as a preposition ; it is used as the name of a word, we mean *the word* "*of*" is to be parsed as a preposition.

§ 3. Ambiguity in Terms.

Few words are quite free from ambiguity. Whenever a word comes into common use, it is certain to have its meaning stretched in different directions, until at last it almost or quite loses its original application. In the "New English Dictionary," published by the Clarendon Press, six distinct meanings are given for the rather technical word "allotment," and nearly seventy distinct meanings for the much-used word

"box," excluding compounds in each case. In practice this ambiguity is less inconvenient than might have been expected, since the context and the circumstances indicate the way in which we are to interpret the word we hear. "Fat" means different things in the mouth of the physiologist and the printer, and has several colloquial uses besides. But there is little likelihood of misapprehension where the uses are so diverse. The danger comes in where the meanings are much alike, and chiefly differ in the limits assigned. Thus endless confusion arises in Psychology, in consequence of the different meanings (all much alike, but no two quite alike) assigned to such words as *feeling*, *sensation*, *consciousness*. And in Political Economy the same trouble occurs. The economist takes many of his technical terms from everyday language, and the old associations cling to them; various economists discard various portions of the everyday meaning, and however careful we may be, we have great difficulty in seeing in what sense a given writer uses them. *Wealth*, *labour*, *capital*, *money*, are terms of this sort; for all of them are unfortunately common to the science of Political Economy, and the usage of everyday life.

§ 4. Connotation and Denotation.

General terms or class names have a kind of double meaning. Primarily they denote a certain undefined number of objects to which they apply. This table,

that table, and the table in the next room, and so on, are part of the *denotation* of the word "table." The denotation means all the things, whether regarded as individuals or grouped into smaller classes (such as library table, dining table, etc.,) to which the name applies. But all these things have certain attributes in common, the possession of which gives them a right to the name. These attributes taken together form the *connotation* of the term. This connotation may be regarded as a sort of secondary meaning.

A proper name has only denotation. There are, as we shall see, no particular attributes necessarily implied by the use of a proper name. When we use a word like *table,* we imply that such a list of implied attributes has been ascertained or could be ascertained; and that anything which had them all would be a *table.* No such list could be drawn up with regard to any proper name. In fact, not a single attribute could be named as necessarily implied. Neither human origin or sex is *implied* by any proper name, however thoroughly it may in practice be confined to human beings or to one of the sexes. Dartmouth does not *mean* the town at the mouth of the Dart; if it did, the town at the mouth of any colonial or American stream called the Dart, would necessarily be called Dartmouth. We must distinguish between:

(1) The list of attributes implied by the use of the name, which being present confer a right to the name.

(2) The attributes merely suggested by a name,

forming part of what I mean, or you mean by it, but not a part of what everybody means.

(3) The attributes which first suggested that the name should be given.

Now the term connotation is used for only the first of these three. The wart on a man's nose, however insistent, is not a part of the meaning of the man's name. Nor is it really implied by the name "Jubilee," which many foolish parents inflicted on their defenceless children in 1887, that the child was born in that year. If it were, all children born in that year would be properly called Jubilee.

When Mill revived the use of the terms denotation and connotation, he used them somewhat differently. "A connotative term is one which denotes a subject, and implies an attribute. By subject is here meant anything which possesses attributes" ("Logic," Book I. chap. ii. § 5). Subject need not be a thing, but may be itself an attribute. Accordingly, simple abstract terms have no connotation, only denotation. (See § 5 below.) In fact, Mill has in view two distinctions which are by no means parallel, viz.: (1) between things and the attributes that distinguish them; and (2) between the primary and secondary meanings of terms. The former seems much the more important distinction.

It is sometimes said that the "connotation and denotation of a term *vary inversely*." This is only a loose way of saying that ordinarily as the connotation decreases the denotation increases, and that as the connotation increases the denotation decreases, and *vice versâ*. To

add fresh attributes to a term is to diminish the number of things to which the term is applicable. And if we wish a term to be applicable to more things than it applies to, we must remove some of the attributes. If we want to fill up the ranks of a regiment we must remove restrictions as to age, or height, or girth of chest; at each step an addition to our possible recruits will be made. But the mathematical expression, "vary inversely," should be avoided, or at any rate carefully explained. No real quantitative relation of the kind really exists. (1) By adding one attribute we may greatly diminish our denotation, *e.g.*, by adding *rational* to *animal;* and by adding another we may make very little change, *e.g.*, by adding *less than eight feet high* to *human being*. If we add other attributes we make no difference at all. Thus if we join to *man, organic*, or *under twelve feet high*, we include just as many individuals as before; but here the new attribute is either a part of the connotation, or follows from it.

(2) Again, if we diminish the number of *individuals* in a class we do not increase the connotation, or *vice versâ*. If a plague decimates the world, the connotation of the term *man* is not increased, nor has the connotation of *Englishman* decreased since the beginning of the century though the population has trebled itself. But if we take several classes which are included under a general term and wish to exclude one of them we can do so by adding an appropriate attribute to the connotation. If we want to make a club more

select, we do so by adding such attributes as *gentlemen having independent means*, *having a university degree*, *holding a commission from her Majesty*, etc. Each of them excludes a number of classes and therefore many individuals.

The words Extension and Extent are often employed as equivalent to denotation; and Intension, Intent, and Comprehension as equivalent to connotation.

§ 5. Concrete and Abstract Terms.

Names of things are called *concrete* terms. Names of attributes or relations are called *abstract* terms. Thus, *Man* is concrete and *Manliness* is abstract. *Man* is a name which can be applied to, and asserted of, an object having a more or less independent existence. *Manliness* can be applied to only a quality or group of qualities, regarded for the purposes of thought as existing apart from an object.

It will be seen that adjectives are concrete terms. They are directly applicable to individual objects. When I say that "John is *manly*," I assert that the term *manly* is applicable to John. If by a further effort of mental analysis I dissociate the quality entirely from the actual being who possesses it, and speak of it as existing by itself (which of course it never does), I arrive at the abstract term *manliness*. This mental process is an instance of what is called abstraction; the attention is drawn away from (*abstrahere*) everything but the quality, and fixed on that only. All

abstraction, however, does not give us abstract terms ; since abstraction is implied in the formation of all general notions. But while all general notions are abstract, the names of general notions are usually concrete. Thus *man*, *secretary*, *the government*, *the British constitution*, are terms which correspond to general notions, produced by abstraction ; but they are not abstract terms, because they are names of things, not of qualities.

It is well to regard abstract terms as purely connotative and as having no denotation, since they are not the names of things. In this, though we differ from Mill and most logicians, we have the authority of Professor Fowler on our side ; and the view avoids many difficulties. According to Mill, most abstract terms denote an attribute, and connote nothing. Here by denotation he simply means primary meaning ; and by connotation, secondary meaning. Such abstracts as are the names of attributes which imply other attributes he calls connotative. The correctness of this, however, is questioned by Dr. Keynes, who accepts Mill's main view, that most abstracts are purely denotative.

The beginner must be cautioned against assuming that all words that may be used as abstracts are always so used. Many words (*e.g.*, *colour*, *action*, *relation*) originally abstract are now most frequently concrete.

Names of states of consciousness, or elements of such states, are concrete. They are phenomena of mental life regarded as having individual existence, and not

as being mere attributes of a concrete mind. Names of the attributes of such states are of course abstract. Thus, *pleasure* is abstract, *a pleasure* is concrete; *volition* is abstract, *a volition* is concrete.

§ 6. Singular and General Terms.

A *singular* (or *individual*) *term* is one which can be affirmed in exactly the same sense of only a single thing. A *general term* is one which can be affirmed in exactly the same sense of an indefinite number of things. First, note that by *thing* here, we mean any object of thought which can be conceived of as having an individual existence, whether in the world of perception or in the world of imagination. It does not include mere abstract qualities or relations.

Proper names, whether consisting of one word or several, are singular terms; *e.g.*, *Moses*, *Romola*, *Susan Smith*, *Weston-super-Mare*. The fact that there are, and have been, many people called John does not make John a general name. For the sole meaning of the term is to identify the person referred to. It happens that other persons bear the same distinguishing mark; just as it may happen that several people have the same unmeaning bands or circles painted on their luggage for the purpose of identification. But there is no suggestion that we can draw up a list of qualities by which we can identify a John whenever we see him. Proper names, to use other language, have no *connotation*, as we have already explained.

They are "arbitrary and unmeaning verbal signs "whose sole province is to indicate the individual object of which they are the name." When a mother speaks of "Mary," she means one particular individual; and no other will serve her turn.

Some singular terms are *significant;* they belong to the class of connotative names. These will be found in all cases to consist of two or more words, one of which at least will be general and connotative. *My house, this pen, the present Prime Minister, the founder of the Romanoff dynasty,* are examples of such significant or connotative singular terms.

A *general term* is one which can be applied in the same sense to any of any indefinite number of things. It implies the existence of an attribute or of a number of attributes, and is applicable to any object which possesses that attribute or group of attributes. It suggests the existence of an actual or possible class of things (whether real or imaginary) which possess these attributes. Thus, *dog, leaf, hearth* are general terms. Such words are sometimes called *class names.*

Adjectives are also general terms since they may be applied with equal propriety to many different things. So are *substantial terms* or names of substances. *Water* is not a singular term, though it looks like one. Thus, when we say "Water forms seventy per cent. of the human body," or ask for "a glass of water," we do not refer to the whole volume of H_2O which exists, but to some of it. The essence of a singular term lies in its individuality or indivisibleness. To speak of "some

Gladstone" or "some London" is absurd. I may indeed speak of some *part* of London; but the term I am using is now frankly general, for "part of London" is applicable to many separate places. You cannot divide an individual without destroying it; that is the very meaning of the word.

§ 7. Collective and Distributive Terms.

Some concrete terms are names of things which consist of a collection of *similar* units. Such terms, *e.g.*, *jury*, and *army*, are called *collective*. *The Alps*, and *the British House of Commons*, are singular collective terms; while the words just given are general as well as collective. But I may use these terms in a distributive sense as well. Thus, if I speak of "the British army" as one army amongst many, an individual in the class of things called *armies*, I use the term distributively. And if I use any general term in the plural number with some such expression as "All taken together," I make it collective. Thus, in "All the freemen (taken together) form the council," freemen is a collective term; but in "All freemen are voters" (= every individual freeman is a voter), the term is distributive. As Dr. Keynes remarks, "The peculiarity of a collective name is that it can be used collectively in the singular number, while other names can be used collectively only in the plural number."

A term is not collective if it is merely the name of

a thing made up of different parts. A collection of like units is implied. And these units are not regarded as individuals included in a class, but as jointly forming the individual. The *books* are not the individuals, forming the class *library*, though they are the units forming an individual library.

§ 8. Relative and Absolute Terms.

Concrete terms may be *relative* or *absolute*. A *relative* term is one which implies the existence of another object, in addition to that which it denotes, which second object receives a name from the same set of facts as form the ground of the first name. *Husband* and *wife*, *subject* and *sovereign*, are correlatives. Thus, a relative term is one which implies a distinct correlative. An *absolute* term is one which does not.

All things are relative; nothing exists in and for itself. Thus, in a sense, knife implies fork, and more obviously, man implies woman. But a man may conceivably exist without the existence of any woman, and *vice versâ;* while a husband could not exist as a husband but for the existence of a wife. This example, however, will serve to show that the line between relative and absolute terms is not easy to draw with precision. The *continued* existence of the thing of which the correlative term is the name, cannot be said to be implied; for *parent* and *child* are undoubtedly

correlative terms, and a child is still a child of his parents even although they may be dead. But the existence of a child implies that at some time or other a parent existed.

The relative term not only implies the existence (at some time or other) of something other than that of which it is the name; the two things must both have names which imply the relation. Relative terms may be adjectives or substantives, but must, in accordance with the definition, be concrete.

Correlative terms would be a better name than relative.

§ 9. Positive, Negative, and Privative Terms.

Negative terms are those which have a negative particle, and which imply the absence of a quality, which the corresponding positive possesses. Negative and positive are strictly relative to each other. If we take *dependent*, *independent*, *un-independent*, the second is negative in reference to the first, and positive in reference to the third. As far as mere meaning goes, *independent* is just as positive as *dependent*. They are mutually contradictory. But the presence of the negative prefix determines which of the two we shall call negative.

In formal strictness the negative term applies to all things which have not the quality or qualities which form the connotation of the positive. Thus, every-

thing which is not *blue* comes under the denotation of the term *not blue.* Not only visible things, but sounds, perfumes, virtues, political constitutions, and theological doctrines are (in the usage of formal Logic) *not blue.* In ordinary usage this is of course not true. We imply a limited sphere of meaning, and tacitly assume that only those things are *not-blue* which might conceivably be *blue.* We restrict the denotation of the negative to things which belong to the class of coloured objects.

The connotation of *not blue* thus becomes a reality. *Not blue* in the strictly formal sense has practically no connotation; for we have no concept which includes only one attribute, namely, the absence of blueness. Its denotation is almost infinite; for it includes everything but blue things. Such terms are, in fact, called *infinite* or *indefinite* terms; they are mere formal negatives.[1] But when used with the limitation imposed by ordinary usage the connotation contains positive elements, and becomes thinkable.

Privative terms are those which imply the absence of an attribute in a subject which usually possesses it. Thus *blind* and *deaf* are privative terms. Older logicians use the term as equivalent to negative. Most privative terms are, however, positive in form, as, for instance, those just given.

[1] It is only fair to say that "infinite" appears to have been applied to them through an error made by Boethius in translating Aristotle. It has, however, been sanctioned by Kant.

§ 10. Contrary and Contradictory Terms.

Contradictory terms are "mutually exclusive and at the same time collectively exhaustive in denotation" (Welton). *White* and *not white*, *good* and *not good*, are contradictories. Formal contradictories are in fact pairs of terms, one of which is positive and the other negative. They not only exclude each other; but between them occupy the whole universe of thinkable things.

Contrary terms are those which express the greatest possible difference within an implied class of objects of thought. They are therefore mutually exclusive, but not collectively exhaustive. There is room for an intermediate class or classes. Thus, *black* and *white*, *good* and *bad* are contraries; because there are things in the universe which are neither black nor white; neither good nor bad.

CHAPTER III.

CATEGORICAL PROPOSITIONS.

§ 1. Judgments and Propositions.

A JUDGMENT is a mental assertion. In a judgment two ideas are brought together; but the synthesis is different from the synthesis which takes place when we form a new complex idea. "This house is red" means something different from "The red house;" and the assertion is the distinguishing feature. It is this which is expressed by the copula.

A judgment expressed in words is called a proposition.

§ 2. Categorical and Conditional Propositions.

A Categorical proposition is one which makes a simple unconditional assertion. A conditional proposition is one which makes an assertion subject to an expressed condition.

Conditional judgments are of two kinds. Hypothetical (*e.g.*, "If A is B, C is D"), and Disjunctive (*e.g.*, "A is either B or C").

Categoricals may be *pure* or *modal.* To the former class belong those which are made simply without any word expressing the degree of belief we have in our statement. To the latter belong those which contain such words as *probably*, *possibly*, *certainly*, which indicate our degrees of belief. Formal Logic is exclusively occupied with the former; and the treatment of Modality is best assigned to Metaphysics which investigates the nature of necessary judgments, and to Mathematics which deals with probable judgments.

§ 3. Quality and Quantity of Categorical Propositions.

By the *quality* of a proposition is meant its character as affirmative or negative. An affirmative proposition asserts that the subject and predicate are in some degree compatible. We do not assert that they are absolutely identical. If I say " S is P " I do not mean that *no* difference exists between S and P, but I assert that S has all the attributes implied by P, and therefore comes under the denotation of P. A negative proposition asserts that the subject and the predicate are in some degree incompatible. The subject is without some of the attributes which are connoted by the predicate; and is not therefore to be brought into the class denoted by it. " In a Negative Proposition we do not deny that the subject has *any* of the attributes connoted by the Predicate, we only deny that it has them all " (Welton).

By the *quantity* of a proposition is meant the extent to which the predicate is asserted of the subject. If the predicate is affirmed or denied of all the things included in the denotation of the subject, the proposition is called *universal;* and the subject is said to be *distributed.* If however we are expressly in doubt whether the predicate applies to all or only a part of the denotation of the subject, the proposition is called *particular;* and the subject is said to be *undistributed.*

The following table exhibits the classification of categorical propositions.

Affirmative	Universal . . .	All S is P.	A
	Particular . . .	Some S is P.	I
Negative	Universal . . .	No S is P.	E
	Particular. . .	Some S is not P.	O

The letters placed in the last column are used to symbolize the four classes. They are said to be taken from the words *AffIrmo* and *nEgO.*

Particular propositions are not to be understood as referring to "some *only*" of the things denoted by the subject. By "some" is meant "some at least," "not none" (cf. Latin *nonnulli*), and "some" may thus include "all."

And it should be noticed that "All S is not P," usually means that "Some S is not P;" it is commonly an O, not an E proposition. This ambiguous form should be avoided. "All the men did not behave badly" means "Some of the men did not behave badly;" and it suggests, though it does not assert,

that "Some of the men did behave badly." This innuendo the logician cannot take into account, because it is only an innuendo. In any case the proposition does not mean "None of the men behaved badly."

The distribution of the subject is expressed in the properly stated proposition. The distribution of the predicate is a matter of inference, since no mark of quantity is prefixed to the predicate. Logicians always assume that the predicate of an A proposition is undistributed. If I say "All dogs are animals," I clearly do not imply that dogs make up the whole class of animals, and that no other animals exist. And this is the case with most A propositions; though there are just a few wherein we may, from special knowledge of the subject matter, take the predicate as distributed, as for instance, "All equilateral triangles are equiangular triangles." The same thing is true of an I proposition. The predicates, therefore, of affirmative propositions are *undistributed.*

The predicates of negative propositions are distributed. For if we exclude every S from P, we at the same time exclude every P from S; "No S is P" implies the absolute separation of the two classes of things. In the same way, if we exclude "Some S" from the class P, we exclude it from P *entirely.* If the portion of S, however small or however large, to which we refer is to be removed from the P's it must be removed entirely, whether *other* portions of S be included in P or not.

§ 4. Infinite, Exceptive, and Exclusive Propositions.

An *infinite* proposition is one which has for its predicate an infinite or indefinite term (see Chap. II. § 9), *e.g.*, "All S is not-P." This is affirmative in form, and must be treated as such.

An *exceptive* proposition is a universal to which some exception is indicated. If this takes a definite shape, *e.g.* "All S is P, except the S which is X," we can retain the universal form, thus, "All S which is not X is P." "All peers except Irish peers are eligible," becomes "No peers in the Irish peerage are eligible." But with this is bound up the statement that "All other peers are eligible." If the exception is indefinite, that is, does not assert what constitutes the class which is excepted ("All the citizens except ten were present"), the proposition must be regarded as particular ("Some citizens were present," and "Some citizens were not present"). Exceptive propositions then are complex propositions, really requiring two or more simple categorical propositions to express them. Such complex propositions are called *exponible* propositions.

Exclusive propositions are also exponible. These are propositions in which the predicate is stated to apply only to the subject. "Only S is P" means "Some S is P," and "No not-S is P." From this latter can be inferred by conversion and obversion that "All P is S" (see Chap. VIII. §§ 2, 3), and from

"All P is S," it follows by conversion that "Some S is P." Hence "No not-S is P" implies "Some S is P," and as exclusion is of the nature of negation, the true force of an exclusive proposition is better given in this form "No not-S is P."

Logic does not deal with numerical statements. "Six of the labourers were elected," becomes for the logician, "Some of the labourers were elected." He leaves to the mathematician the manipulation of all statements of exact numerical quantity; reserving to himself only propositions which make assertions of *all*, or *none*, or *some* (not-none) of the members of a class. *Most* really means "more than half," but as the conception of "half" is purely arithmetical, we must replace "most" by "some," thus somewhat weakening the meaning. Propositions in which the quantity is indicated by "most," are sometimes called *plurative.*

The word *few* requires a word or two of explanation. If we say "*A few* members of the council are tradesmen," we mean somewhat less than half; and for logical purposes we must replace this by "some members." If we say "*Few* men are absolutely truthful," we do not mean quite the same thing by "few." We mean "Not many men are absolutely truthful;" and this will appear as an O proposition, "Some men are not absolutely truthful."

§ 5. Opposition of Categorical Propositions.

We see then that all categorical propositions have

to be reduced to one of the four forms, A, I, E, O, in order for the logician to deal with them. It is now necessary to see what are the relations between these four types of proposition.

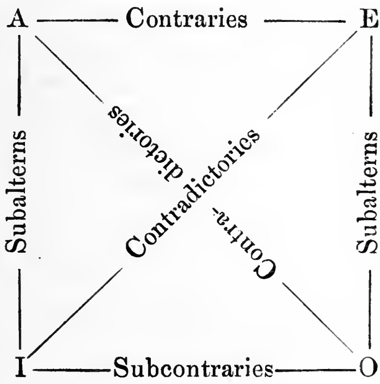
(1) Clearly, if a universal proposition is asserted, we imply the truth of its corresponding particular. If "All candles have wicks," then "Some candles have wicks;" if "No books are made of iron," then "Some books are not made of iron." The particulars I and O are said to be the respective *Subalterns* of the universals A and E, with the same subject and predicate; A and E being called the *Subalternant* propositions. The subalterns differ from their respective subalternants, then, only in quantity, and not in quality.

Clearly, too, the truth of the particular does not necessarily involve the truth of the corresponding universal. On the other hand, of course it must not be understood to deny the truth of the corresponding universal. In everyday usage if one says, "Some criminals are not insane," one is often understood to mean that you believe that some are insane, and that you disagree with the statement that "No criminals are insane." In the same way, to say that "Some Pharisees are honest"—with a certain emphasis on the some—is taken to mean that you do not think *all* are honest. This innuendo—which is not obvious in the written language—must be disregarded by the logician who must take the words in their plain grammatical meaning.

(2) Propositions of opposite quality but the same quantity are either *Contraries* (A and E) or *Sub-contraries* (I and O). Contraries cannot both be true and may both be false. Sub-contraries may both be true and cannot both be false. "All men are liars," and "No men are liars" will serve to exemplify the former; "Some men are black-haired," and "Some men are not black-haired" will illustrate the latter.

(3) Propositions of opposite quality and opposite quantity are *Contradictories* (A and O; E and I). Of these both cannot be true and both cannot be false; that is, one must be true and the other false. If we assert that every X has a certain attribute, we deny that some X's have not this attribute, and *vice versâ.*

These relations are most easily kept in mind by the following diagram, called the *square of opposition,* which explains itself:



From this we can see what is meant by saying that statements are *diametrically* opposed to each other, namely, that they are contradictories to each other; because in the table of logical opposites the contradictories are placed at the opposite extremities of the diameters, or diagonals. And we shall guard against calling contraries or subcontraries having the same subject and predicate the *converse* of each other; this term, as we shall see, has quite a different sense.

§ 6. Analytic and Synthetic Propositions.

An *analytic* judgment is one in which the predicate has the same connotation, or part of the same connotation as the subject. A *synthetic* judgment is one in which the predicate does not form part of the connotation of the subject. Thus, "the German Emperor is a sovereign prince" is an analytic judgment; "the German Emperor is a yachtsman" is a synthetic judgment. The first proposition could be inferred by anybody who knows the meaning of the subject; the second could not.

It is sometimes said that propositions originally synthetic become analytic when they have been once made. "To us, with a large experience of fire, the proposition 'fire burns' is analytic—simply a verbal proposition: what we mean by 'fire' is a subject which among other predicates has this of burning; the burning is an integral part of our concept. But to a child whose experience of fire is less, whose con-

ception includes brightness and form but not burning, the addition 'it burns' would be as such a synthesis" (Lewes).

But Logic assumes that men mean the same thing by the same words; that "fire" has a definite connotation which can be assigned. The imperfect apprehension of this connotation by a child does not concern us. For those who use the term "fire" correctly, burning is a part of the meaning. The only question is, does the predicate form part of this authorized connotation of the subject? If not, the proposition is not analytic.

In practice of course the convention breaks down. All sovereigns are supposed to contain exactly the same number of grains of gold; and this fiction is necessary to commerce. As a matter of fact they vary appreciably within certain fixed limits. So it is with the connotation of words. Although men do not invariably agree in the connotation of words, a necessary convention assumes that they do, until attention is called to the divergence.

Analytic propositions are often called *verbal,* and sometimes *explicative* propositions. Synthetic are also called *real* or *ampliative.*

§ 7. Predication.

Several interesting questions arise with regard to propositions. Of these the chief are:

(1) About what is predication made? What exactly

is the meaning of the subject and predicate of a proposition?

(2) What is the real nature of the copula, what exactly do we mean by saying "S *is* P?"

(3) And as a special case, involving considerations of both kinds, is real existence always, or ever, implied in saying "S is P?"

(1) The ordinary man would say that when we make an assertion, *e.g.*, "The sky is blue," we refer to real things, and express a relation which exists in the objective world. And in this he would have the authority of Mill and other acute thinkers on his side. At the same time it may be urged, that after all no judgment of mine can really carry me beyond the circle of my own ideas, and when I say that "The sky is blue," I can only mean that my idea of the sky is inevitably accompanied by or associated with the idea of blue. If Knowledge be, in the words of Locke, "Nothing but the perception of the connection and agreement or disagreement and repugnancy of any of our ideas," the propositions which express it must be expressions of the same "connection and agreement or disagreement and repugnancy."

A third view has been taken by Hobbes and other Nominalistic thinkers. These writers hold that the only relation actually predicated is a relation between the names. "Truth," says Hobbes, "consisteth in the right ordering of names in our affirmations." And in another work he says, "It is true (for example) that *man is a living creature*, but it is for this reason, that

it pleased men to impose both these names on the same thing." This view is obviously insufficient. When I say that "The sky is blue," I certainly mean something more than that the term "blue" is applied to the same thing as the term "sky." I mean more even than that my ideas of "sky" and "blue" are in "connection and agreement;" I distinctly mean that the real thing sky has the real quality blue. I assert that the connection and agreement exist for all minds and not only for mine.

A true proposition thus expresses an objective relation, something which is true for everybody. As a matter of fact, my ideas may imperfectly correspond to facts; and my words may imperfectly conform to ordinary usage. But when I make a proposition I assume the correspondence between words, thoughts, and things. I assume that I mean the same thing as other people by the words, that my use of the words and my ideas correspond with those of everybody else, and that my words express a relation which would exist for anybody and everybody, and (perhaps) would exist even if there were nobody to perceive it.

(2) There are several ways of regarding the relation of the predicate to the subject which is indicated by the copula. The principal are:

(*a*) *The predicative view.* The subject is understood in denotation, the predicate in connotation; and the assertion made is that the thing named by the subject possesses the attributes connoted by the predicate. This is obviously the most natural way of regarding

the proposition when the predicate is an adjective; but as Mr. Welton points out, it is equally applicable to the case when the predicate is a substantive. It only fails where both terms are proper names, *e.g.*, "Tully is Cicero."

(*b*) *The class-inclusion view.* Both subject and predicate are read in denotation; and the meaning is that the class S is included in the class P, or excluded from it. This way of looking at the relation of S and P undoubtedly does not represent what is actually in the mind; it is psychologically false. But it is extremely convenient for purposes of inference, and lies at the bottom of the Aristotelian account of the processes of inference. A further development of it leads to

(*c*) *The equational view*, which by declaring the quantity of the predicate as well as that of the subject, assimilates the proposition to an equation. "All dogs = some animals," or as Jevons prefers to write it, "All dogs = dog animals."

(*d*) *The attributive view.* This is accepted by Mill. Where the subject is connotative the meaning of a proposition is, that "whatever has the attributes connoted by the subject has also those connoted by the predicate; that the latter set of attributes *constantly accompany* the former set" (Mill, Bk. I. chap. v. § 4). This view again is psychologically false, and is practically inconvenient. Besides, it is unworkable in the case of particular propositions. Mr. Welton remarks that if we are to interpret the subject in connotation, in the proposition "Some birds are web-footed," the

subject must mean "Some attributes of birds.' Directly you leave universals you must consider the denotation of the subject.

§ 8. Predication and Existence.

(3) The question as to whether real existence is implied in propositions turns to some extent on the implication of ordinary colloquial language, and to a large extent on the definition we should give of "real existence." Dr. Venn and Dr. Keynes hold that universals do not in ordinary speech imply the existence of the subjects, and that particular propositions do.

This, however, is very doubtful and the simplest plan is to hold with the older logicians, such as Mansel, that no real existence is necessarily implied in any proposition. Existence in some sense—in the world of imagination, or of conception—is no doubt implied; but this is not what is meant by real existence. The classes with which Logic deals are only mental products, and do not profess to have a further reality in the sphere of perception and action. If I say "Koot Hoomi is a Mahatma," I am implying the objective existence of Koot Hoomi and of Mahatmas only to this extent, that persons interested in what is called occult research will be able to form some mental image or notion answering to my words. But I do not imply that either exists in the world of sense perception. It is true that if I say "The Duke of

Cambridge is Commander-in-Chief," I certainly do in ordinary usage imply that the things denoted by subject and predicate actually exist in the world of sense perception; but this additional implication of existence in the world of perception adds nothing to the possibilities of inference. Whatever I can do with the second proposition I can do with the first; and there is absolutely nothing in the form to indicate any difference of meaning.

Real existence, then, must be specially predicated or denied. We must say "The Duke of Cambridge exists," "Koot Hoomi does not exist." Such propositions are called *existential propositions.*

CHAPTER IV.

CONDITIONAL PROPOSITIONS.

§ 1. Hypothetical Propositions.

CONDITIONAL propositions are of two kinds, as already said. Hypotheticals are those which explicitly express a condition, *e.g.*, "If A is B, C is D." Disjunctives express the condition in a less explicit form, by asserting that one at least of several alternative predicates applies to the subject, *e.g.*, "A is either B or C." What is really asserted in either case is the dependence of the truth of one proposition on the truth of some other proposition or propositions.

"If the saucepan is dirty, the milk is liable to burn," is in its essence a statement that one may infer the milk's liability to burn from the dirtiness of the saucepan. The former statement is called the antecedent, the latter the consequent. Hence a hypothetical proposition cannot be expressed quite adequately as a categorical proposition. There is always some loss of meaning. We can no doubt say, "Milk boiled in a dirty saucepan is liable to burn;" but we do not mean *quite* the same thing as is expressed by the hypothetical. We no longer imply that the boiling in a dirty saucepan

is the ground or cause of the liability to burn. Still the difference of meaning is so slight that it may be disregarded, and we may at once allow that most hypotheticals can be reduced to a categorical form.

There is, however, one important type of hypotheticals which requires special consideration. Several recent writers on Logic have pointed out that there is a real distinction between true hypotheticals which really express a dependence of the truth of one proposition on that of another, and quasi-hypotheticals (sometimes called "Conditionals," a word here used in a new sense) which only express the dependence of one fact on another. There is often no difference in the *form*, but there is a great deal of difference in the meaning. The true hypothetical can only be *adequately* written in the form, "If P is true, Q is true," *i.e.*, "If the proposition A is B is true, the proposition C is D is true." While the quasi-hypotheticals ("Conditionals") may be written in the ordinary way, "If A is B, C is D," or even expressed in a categorical form, thus, "Cases of A being B, are cases of C being D," although something is lost in the meaning.

To put this in another way. The dependence of the consequent on the antecedent may be due to (1) a purely formal and logical relation, or to (2) a material relation between objective phenomena. "If all dogs have tails, your dog has a tail," is an expression of the former kind ; it is a shortened piece of reasoning, what is known as an enthymeme (Chap. X. § 5).

Therefore it is a true hypothetical. Our previous example of the dirty saucepan is a hypothetical of the second type, which we have called quasi-hypotheticals. The dependence of the consequent on the antecedent is due to certain material facts, and is not purely formal. We can assign a group of facts as the basis of our potential inference ; in the other case we cannot.

This distinction has been accepted by Dr. Keynes, Dr. Venn, and Mr. Welton, as well as other logicians. It is, however, perhaps less fundamentally important than is sometimes asserted.

There is nothing in the mere form to distinguish the formal hypothetical (hypothetical proper) from the other type. Both assert the dependence of the consequent on the antecedent. For the purposes of inference, though not without some loss of meaning, in the case at any rate of formal hypotheticals, they may both be written in a categorical form.

Hypotheticals will take one of the following four forms : (*a*) If A is B, A is C ; (*b*) If A is B, C is B ; (*c*) If A is B, B is C ; (*d*) If A is B, C is D. In (*a*) and (*c*) we have the conditional statement of a syllogism with the major premise omitted ; (*b*) is a syllogism with the minor premise omitted ; (*d*) implies a larger train of reasoning than can be expressed in a single syllogism.

Type (*c*) may be expressed categorically thus: "A B is C." The other types can be expressed thus : "Cases of A being B, are cases of C being B," etc.

§ 2. Disjunctive Propositions.

A *Disjunctive* proposition is one which asserts of the subject that one or other of certain alternative predicates applies to it; *e.g.*, " S is P or Q," " Human beings are either male or female."

The point of chief importance in connection with disjunctives is the question whether the alternatives are necessarily exclusive. " S is P or Q " means that " if S is not P, it is Q," and " if S is not Q, it is P." It thus contains at least two hypotheticals. But does it also contain the implication that if S is P it is not Q, and if S is Q it is not P ? In practice, mutual exclusion is certainly not implied. We may happen to know that P and Q exclude each other, and that if S is one it cannot be the other ; but there is nothing in the form which makes this inevitable. When we say that " A man is either a fool or a knave," we certainly do not exclude the possibility of his being both. It is never safe to take the predicate of a disjunctive proposition in the exclusive sense, unless from our acquaintance with the particular subject matter we know that the alternatives never overlap.

Disjunctives, as we have just seen, are expressible by hypotheticals, and can therefore be expressed by categoricals, in so far as the component hypotheticals can.

A distinction parallel to that between hypotheticals has been made between Disjunctives. Those that reduce to purely logical or formal hypotheticals have been

called Contingent Disjunctives ; while those that reduce to material hypotheticals have been called Divisional Disjunctives.

The following scheme represents the division here adopted:

$$\text{Conditional propositions}\begin{cases}\text{Hypothetical}\begin{cases}\text{Formal}\\ \text{Material}\end{cases}\\ \text{Disjunctive}\begin{cases}\text{Contingent}\\ \text{Divisional}\end{cases}\end{cases}$$

§ 3. The Opposition of Conditional Propositions.

Since conditional propositions are to some extent susceptible of distinctions of quality and quantity we can draw up a scheme of opposition from them :

(1) *Hypotheticals* of the purely formal type are regarded by Keynes, Welton, and others, as not susceptible of distinctions of quantity. They are all singulars. "'*If* A, *then* C' is contradicted by '*If* A *then not* C' for the consequence must either follow or not follow from the antecedent; there is no intermediate choice" (Welton). The only possible opposition is between these two forms.

Hypotheticals of the material type (called by Welton, Conditionals) admit of distinctions both of quality and quantity. The four forms corresponding to A, E, I, O, among categorical propositions are thus given by Welton ("Logic," i., p. 277) :

"If any S is M, that S is always P" (A).

"If any S is M, that S is never P" (E).

"If an S is M, that S is sometimes P" (I).

"If an S is M, that S is sometimes not P" (O).

(2) *Disjunctives* are all affirmative. As we have just seen, they give a choice of predicates; one of which must be capable of being affirmed of the subject. To contradict a disjunctive we must deny *all* the predicates. The negative of "S is P or Q" is "No S is either P or Q," that is, "S is neither P nor Q." This is a categorical proposition, though it has at first sight the appearance of being disjunctive.

Disjunctives, at any rate those called by Welton Divisional Disjunctives, are susceptible of differences of quantity. Thus "Every S is either P or Q" answers to an A proposition among categoricals; and "Some S is either P or Q" answers to an I proposition. The negative of the former will be the categorical "No S is either P or Q" (E); the negative of the latter will be, "Some S is neither P nor Q" (O). The last proposition will be the contradictory of "Every S is P or Q;" and the universal negative "No S is either P or Q" will be its contrary.

CHAPTER V.

THE PREDICABLES AND CATEGORIES.

§ 1. The Predicables.

The predicables as originally understood were the four possible relations in which the predicate might stand to the subject in regard to denotation and connotation.

When the predicate agrees with the subject in connotation (and therefore also denotation), the predicate is a *definition* of the subject. Where the connotation of the predicate forms part of the connotation of the subject, the former is a *genus* of the latter. Where the predicate forms no part of the connotation of the subject, but is necessarily connected with it, the predicate is called a *proprium* (or property) of the subject; they will have the same denotation, since wherever the set of attributes connoted by the subject is found, that connoted by the predicate must also be found. When the predicate differs both in connotation and (to some extent in) denotation from the subject, the predicate is called an *accidens* (or *accident*) of the subject.

The following table is slightly altered from Mr. Welton (vol. i., p. 95):

Predicables	I. Entirely agreeing in denotation with the subject	1. Agreeing entirely in connotation .	*Definition*
		2. Connotation inferrible from or necessarily connected with that of subject . . .	*Proprium*
	II. Partially agreeing in denotation with the subject	3. Agreeing partly in connotation . .	*Genus*
		4. Wholly differing in connotation .	*Accident*

Propositions affirming a proprium or an accident are synthetic; the others are analytic. "Man is a two-legged mammalian animal possessing reason" is a definition; "Man is a mammalian animal" is a declaration of genus; "Man is a cooking animal" is a declaration of a proprium; "Man has thirty-two teeth" is a declaration of an accident.

Later on a five-fold scheme of the predicables was accepted: viz., *genus*, *species*, *differentia*, *proprium*, and *accidens*. Definition gives places to species, and difference is added; while the predicables lose to some extent their original place as a division of the possible relations of predicate to subject.

As found in the mediæval scholastic logicians this doctrine was based on the belief that the essences of things, those assemblages of attributes which lead us to give a name, had a real existence and could be known. The species declared the "whole essence"

of the thing. The world was made like a pigeon-holed cabinet, each class neatly and completely cut off from all others.

In more modern writers there has been a partial reversion to the older Aristotelian doctrine.

A *genus* is a wider class, viewed as containing narrower classes or *species* under it. The terms are strictly relative to each other, so that any class may be a genus to some narrower class, and a species to some wider and less defined class above it. The surplus of connotation found in the species over the connotation of the genus which contains it is the *difference.* The difference distinguishes the given species from other species within the same genus, but not from all other classes whatever, as the scholastic logicians considered. Thus, under the genus Man one can bring the species Black man, etc. ; black is a differentia.

Aristotle had said that difference is "of the nature of genus." This view is accepted by modern writers. In the example just given, "black" and "man" may either of them be regarded as a difference, and either as a genus, of the species "black man." With the scholastics, however, species was taken absolutely ; it was the lowest real class (*infima species*) which could only be divided into individuals, and could not be split up into narrower classes ; and difference was also taken absolutely—as distinguishing this species not only from other species in the same genus, but from all other species whatever. They would not have regarded "black" as a difference, or "black man" as a species.

While a *proprium* belongs to every member of a class, and is necessarily connected with the connotation of the class-name, an *accident* may or may not belong to any member of a class, and is not necessarily connected with the connotation.

A distinction has been made between *inseparable* and *separable accidents.* The former, though not necessarily connected with the meaning of the class-name, not a consequence of any attribute in its connotation, are, as a matter of fact, found in every member of the class (*e.g.*, having thirty-two teeth in the case of man). The latter are attributes which occur in some members of a class and not in others (*e.g.*, having black hair, in the case of man).

A species, it may be added, viewed in relation to the *individuals* contained under it is called a *species prædicabilis;* viewed in relation to the genus under which itself stands, it is called a *species subjicibilis.* Intermediate species between the highest and lowest are *subaltern species* (or genera).

§ 2. Infimæ Species.

If we go forward in connotation we come at last, according to the scholastics, to *infimæ species*, species so fully defined that they can only be split up into individuals. Such a species is Man. All minor divisions (*e.g.*, into black men and white men, or Frenchmen and Englishmen, etc.) are artificial, and are not founded in the nature of things. Modern logicians do

not recognize this distinction. Any sub-class is for them a species compared with the class which contains it.

Something of the scholastic meaning has, however, been preserved in the Natural History use of the word species. The older naturalists thought of species as the lowest class marked out by Nature, and as consisting of individuals having a very large number of essential attributes in common which remained constant from generation to generation. "There are as many distinct species," said Ray (who died in 1705), "as the Infinite Being created distinct forms at the beginning of the world." They gave the name "variety" to a sub-class which differed in a few assignable and unessential points from other members of the species. A genus was a class which included several species; while genera were ranked into families, and then into orders, etc. While these names are kept up, their meaning has to some extent altered. There is no means of determining what characters are essential, since for us "essential" can only mean "included in the connotation of the class-name," except by seeing how the class-name is used by recognized authorities. "One botanist enumerates 300 German species of the common composite Hieracium [Hawkweed], another reduces them to 106, another to 52, another to about a score." Everywhere continuity is recognized in nature, and the pigeon-hole conception has almost disappeared from the text-books. Classification of natural objects is an affair of man's con-

venience; or, as Locke put it—a century and a half before the naturalists woke up to the truth—"the boundaries of species whereby men sort them are made by men" ("Essay," III., vi., 37). Nature gives the points of resemblance, but the "sorting" is done by man. This opinion is fortified by the doctrine of Evolution, which shows us that species and genera, families and orders, are all more or less arbitrary boundaries marked off in a continuous pedigree.

It is strange to find J. S. Mill re-stating, a few years before the publication of the "Origin of Species," the old scholastic doctrine of species; and stranger still to notice that he made no alteration in it after Darwin's book appeared. Mill's "Real kinds" answer to the scholastic genera and species.[1] Each of them is "distinguished from all other classes by an indeterminate multitude of properties." There was no reason to be given for the persistence and constancy of the group of attributes. It was not the work of any cause; it was an ultimate fact of co-existence.

For the modern logician species and genus are relative terms. There are no *infimæ species.* For the process of subdivision is potentially infinite. A class can be formally subdivided into two by applying to it any attribute whatever; A's are either A's which are B's or A's which are not B's. And there is no funda-

[1] See Chap. XX. § 2, below. Compare Venn, "Empirical Logic," p. 83, *sq.*

mental distinction between classes artificially made in this way and natural classes. As we have already said, the classes which Logic deals with are mental products and have no direct reference to the world of reality. Addition of fresh attributes is therefore only limited by the Law of Contradiction. (See Chap. III. § 8.)

In any particular science or branch of study the terms *summum genus* and *infima species* have a meaning. To the zoologist, animal is the *summum genus*, and the thousands of zoological species and varieties the *infimæ species*. To the musician, tones are his *summum genus*, and his *infimæ species* are particular notes of particular quality. But there is no absolute *summum genus*, and no absolute *infimæ species*.

§ 3. The Categories or Predicaments.

In the same way there are no *summa genera*, or highest classes, which can be brought under no higher class; unless indeed "existing things," or "objects of thought," can be so regarded. Any actual class, A, must have a negative not-A; and these two together must be capable of being thought under some higher class, M. There must be something in common between any real A in experience and not-A, or we could never think them as in opposition, the antithesis would have no meaning. This Law of Homogeneity is not indeed valid with reference to purely formal

symbols; but it is true of actual things. (See below, Chap. VII. § 4.)

The scholastics held, however, that there were ten *summa genera,* which they called the *Categories* or *Predicaments.* They are (1) Substance (οὐσία); (2) Quantity (πόσον); (3) Quality (ποιόν); (4) Relation (πρός τι); (5) Action (ποιεῖν); (6) Passion (πάσχειν); (7) Place (ποῦ); (8) Time (πότε); (9) Posture (κεῖσθαι); (10) Habit (ἔχειν). These were regarded as the highest classes within which things could be placed.

They seem to have been originally intended by Aristotle as "an enumeration of the different grammatical forms of the possible predicates of a proposition viewed in relation to the first, Substance, as a subject." As Mansel says, Aristotle's Categories are "an enumeration of the different modes of naming things, classified primarily according to the grammatical distinctions of speech, and gained, not from the observation of objects, but from the analysis of assertions." In the list of Predicables he enumerated the possible formal relations of predicate to subject; and now he inquires what are the material relations of predicate to subject, what classes of assertions can be made of any given thing. He, doubtless, began by a partially grammatical classification. In some of his works he omits (9) and (10) of the above list, which are certainly redundant. The last six may in any case be regarded as subdivisions of the fourth, Relation.

The following classification brings out Aristotle's probable meaning.

Substance (*ens per se*)	Substance .	(1)
Accident (*ens per accidens*)	Quantity . .	(2)
	Quality . .	(3)
	Relation . .	(4)

Relation (4) is subdivided into

Place . . .	(5)
Time . . .	(6)
Action . . .	(7)
Passion . .	(8)
Posture . .	(9)
Habit . . .	(10)

By Passion (8) is of course meant the fact of enduring, or being the object of any activity; for instance, the condition of being struck or being wounded. By Habit (9) is meant possession; for instance, the fact of having dress or sword.

The criticisms of Mill and others have nearly all been directed against the scholastic view of the categories as a classification of *things*, rather than the earlier Aristotelian view of them as a classification of the possible predicates of a given subject.

CHAPTER VI.

DEFINITION AND DIVISION.

§ 1. Kinds of Definition.

ALMOST all terms are equivocal. As precision in the use of terms is absolutely necessary for correct reasoning, the need for a process of definition arises. By definition we mean "the explicit statement of the connotation of a term."

There is no such thing as a definition of a thing. This is only an elliptical expression for the definition of the name of a thing.

A distinction has been made between *Real* and *Nominal Definitions;* but scarcely two logicians agree in their view of it. The distinction is a legacy from scholastic logic, and by modern logicians no clear meaning can be given to it. According to Mill a Real definition is one which "along with the meaning of the name covertly asserts a matter of fact," viz., the real existence of a thing which has these attributes. Hamilton and Mansel use Real definition for the "determination of the contents of a notion," while they reserve Nominal definition for the assignment of

the meaning of a word. Yet what we call a notion is in fact the meaning of a word.

If we assign the full connotation of a term we have a perfect or complete Definition. What is often called an *Imperfect Definition* is not a Definition at all, but a description of a thing sufficiently close for us to recognize it as forming part of the denotation of a given term. An imperfect definition need not contain any of the essential characters which form the connotation; but may consist entirely of propria or accidents.

The *explanation* of a word does not necessarily involve giving its proper connotation. Dictionary "definitions" are usually only explanations, in which some partial synonym of the word is given, or a description of the thing to which the name applies. They usually give the popular connotation, the rough, current meaning, but not the exact list of attributes, all of which must be present, and none of which must be absent, if the name is to be given.

§ 2. Rules of Definition.

These may be given as follows:

(1) The Definition should contain neither more nor less than the essential attributes, that is, the connotation of the term defined.

(2) It should not be expressed in ambiguous, figurative, or unfamiliar language.

(3) It must not contain any term which means the same thing as the term defined.

(4) It should not be negative where it can be affirmative.

If we observe Rule (1) we shall exclude from our definition propria and accidents. A definition which includes these is liable to be too narrow or too wide; it will not apply to all the objects properly called by the name, or it will apply to objects not properly called by the name. We have to assume that the connotation is fixed, or fixable, and that there is no absolute ambiguity in the use of the term. This is not absolutely the case, except with a few scientific terms, and fixing a definition actually takes the form of a discussion as to the matters of fact. Is the attribute x always present in all instances we call by the name N; and should we apply the name if it were absent? The definitions of science are often only provisional and tentative, because these questions cannot be yet properly answered. Impatience to arrive at final definitions will drive a science into barren formalism.

No definition, then, is quite absolute. We understand that the definition is relative to the particular end in view; it gives the meaning of the word as employed in the particular scientific corner in which our work lies. If a term is only available in one given corner (*e.g.*, the terms of chemistry), its definition is comparatively a simple matter.

(2) We must not define *ignotum per ignotius*. Scientific definitions often appear to break this rule when they do not; for it must be remembered that the *ignotius* is relative. During the revolt against

scholasticism in the seventeenth century much contempt was cast on the Aristotelian definition of Motion as "actus entis in potentiâ quatenus in potentiâ." "What more exquisite jargon could the wit of man invent?" asks Locke. "Who is there that has ever learned from it any of the properties of motion?" inquires the "Port-Royal Logic." But these scientific definitions presuppose that the terms used have already been explained; although unfamiliar to the plain man, they have been already discussed by the philosopher. Mr. Spencer defines Evolution as "an integration of matter and concomitant dissipation of motion, during which the matter passes from an indefinite, incoherent homogeneity to a definite coherent heterogeneity, and during which the retained motion undergoes a parallel transformation." But all the terms have been discussed beforehand, and are what the scholastics called *notiora in se*, since they are relatively simpler than the extremely complex idea answering to the word Evolution, though they are not *notiora nobis*—where the *nos* are ordinary folk without special philosophical knowledge.

(3) The breach of this rule is called a *circulus in definiendo*. It is not committed if our definition contains *part* of the term defined; *e.g.*, I may say that "an equilateral triangle is a triangle which has three equal sides." Here the term *triangle*, forming the genus to the species *equilateral triangle*, is supposed to have been already known. But I must not define "life" as "vitality," or "liberty" as "freedom." Definition,

in fact, usually takes place *per genus et differentiam*, that is by assigning a class to which a thing belongs, and then pointing out the specific features which distinguish this thing from others within the class.

(4) The fourth Rule simply means this, that you cannot explain adequately what a thing is by saying what it is not, what attributes the word does not connote. Such a negative process leaves us in a state of uncertainty, and often leads to a breach of Rule (1). Euclid's definition of a point, as "that which has no parts, or which has no magnitude," is open to this objection. An idea or a feeling would come under it as well as a geometrical point.

§ 3. Limits of Definition.

A proper name cannot be defined because it has no connotation; all other singular terms which connote groups of attributes can be defined. We cannot define a term which connotes a single attribute (*e.g.*, a simple abstract term).

We can sometimes, however, substitute a description which will serve many of the ends of definition, by rendering it impossible to misapply the given term. I cannot define redness, but I can describe it as the "quality of colour sensation produced in normal persons by vibrations of ether of the rapidity of 450 billions per second." This gives a proprium of redness; it is not what we *mean* by redness.

Names of natural objects and many complex abstract

terms are difficult to define; because these are apt to be extremely equivocal from the large number of different classes of persons who use them, and it is difficult to settle whether definition will cover all or several of these uses. On the other hand, names of a technical character, especially those recently introduced, have their connotations precisely ascertained.

§ 4. Division.

As Definition is an analysis of the connotation of a term, so Division is an analysis of its Denotation—the assignment, however, not of the individuals, but of the sub-classes into which the given class can be divided.

There are some processes which are occasionally confounded with Logical Division, viz.:

(1) Analysis of a physical thing into its attributes, sometimes called metaphysical analysis; *e.g.*, the enumeration of the different attributes of water, its fluidity, its colour, and so forth.

(2) Analysis of a mathematical or quantitative whole into parts lying outside each other in space, *e.g.*, the enumeration of the parts of a house, floors, ceilings, walls, etc.

The process of logical division is not purely formal, ince the *fundamentum divisionis*, or ground of division of any class to be divided (called the *totum divisum*), is not a part of the definition of the class. If I want to divide the class *living being*, I cannot do it without going outside the connotation to some fresh attribute,

say *sensibility*, which is possessed by some members of the class and not by others. It requires some further knowledge of the things contained in the class than the mere connotation of the class-name affords.

§ 5. Rules of Division.

The rules of Logical Division have been stated as follows:

(1) Each act of division must have only one *fundamentum divisionis.*

(2) The constituent species must exclude each other.

(3) They must, when taken together, be equivalent to the whole genus.

(4) If the division be continued beyond one step, each species must emerge directly from its own proximate genus.

(1) If I want to divide Man, I must take either country, or race, or religion as a basis of division, but not all three together. Otherwise I may get such a division as this: Men are Americans, Frenchmen, Germans, Negroes, Protestants, Catholics, etc. But if I first of all divide in accordance with country, I can at my next step introduce race as a *fundamentum*, and then, at a third step, religion.

(2) The result of employing two or more *fundamenta* at one step is to give what is called a *Cross division*, in which the classes overlap. Thus some Americans are Catholics and some are Protestants; while some

Catholic Americans are negroes, and so are some Protestant Americans. We, however, want only one place for each individual.

(3) If the species do not exhaust the genus, we have not a place for each individual. If we divide Religion into Heathen and Christian, we leave no places for Mohammedanism and Judaism.

Rules (2) and (3) will never be broken if we adopt *division by dichotomy*, by which a class is divided into two species, one of which has, and one of which has not, a given attribute. Thus the class A is divided into A's which are B's (written A B), and A's which are not B's (written A b). If this process of dichotomy is continued a place will be found for every possible thing.

A

A B   A b

A B C   A B c   A b C   A b c

Every A that exists falls into one of the four classes: (1) A's which possess both the attributes B and C; (2) A's which possess only B; (3) A's which possess only C; (4) A's which possess neither.

At the same time, this is not always a very convenient method of division. When we happen to know that the classes are mutually exclusive it is quicker and better to divide thus:

Europeans

Englishmen   Frenchmen   Germans   Men of other races

rather than divide thus:

Europeans

Englishmen — Not Englishmen

Frenchmen — Not Frenchmen

Germans — Not Germans

and so on. But if we do not know whether the classes are mutually exclusive, we must keep to the dichotomous division, subdividing every class. All good division is not necessarily dichotomous; but it must be reducible to a dichotomy. Dichotomy may serve to first indicate our classes, or to test a suspected division; but it is not in itself always a convenient arrangement.

(4) The fourth rule, sometimes expressed thus, "Divisio ne faciat saltum," insures that the species are put as subdivisions of their own genera. It avoids such a division as this:

Religion

Polytheism — Catholicism — Protestantism — Mohammedanism — etc., etc.

where clearly the class Christianity should occupy the place of the second and third classes, to be afterwards subdivided. If a species has a connotation much wider than the genus from which it is immediately separated, we can see that an intermediate class can be found; and a better arrangement would be:

Religion

Polytheism — Monotheism

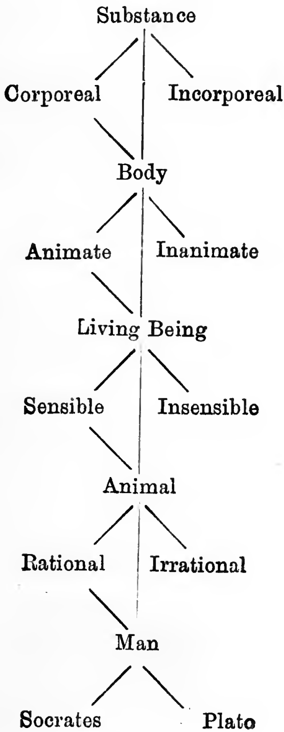
Mohammedanism — Judaism — Christianity

Catholicism — etc., etc.

§ 6. Porphyry's Tree.

Porphyrius, a Neo-Platonic philosopher of the third century, gave an example of complete division by dichotomy, in what has since been called after him the Tree of Porphyry.

It is usually given as below, following St. Thomas Aquinas:



Here we have the *summum genus*, the subaltern genera and species, and the *infima species*, divided into individuals. The vertical line connecting these is called the *linea prædicamentalis*, a series of classes of decreasing generality from the highest to the lowest. The whole arrangement was called by early writers on Logic a Category; but this term is nowadays reserved for the *summum genus*.

CHAPTER VII.

THE LAWS OF THOUGHT.

§ 1. The Laws of Thought.

THESE are the fundamental principles which underlie all thinking. If we think, and do not merely dream or imagine, our thinking must be carried on in accordance with them. Compliance with these principles is necessary for all valid reasoning ; and invalid reasoning is not, in the true sense of the word, reasoning at all—it is a sham which is doing duty as the real thing. They underlie, however, not only all processes of reasoning or inference, but also the processes of judgment and conception, which may be said to precede inference.

The three Laws of Identity, Contradiction, and Excluded Middle, must not be regarded as the results of rational experience, or as due to conscious Induction from facts. They are prior to this, since every valid inference implies them. Reflection on our experience doubtless reveals these principles in explicit form ; but to reason at all we must implicitly recognize them. They are then *logically* prior to all fully conscious knowledge, since they are implied by it. Chrono-

logically speaking, they of course come later. In other words, though the infant assumes them, he does not necessarily formulate them; and most people get quite comfortably through their lives without ever hearing about them. It is only by careful analysis of the processes of thought that we detect these regulative principles underlying them.

They are statements of the necessary conditions of consistent thinking, or as Mill would prefer us to say, of consistent expression.

§ 2. Laws of Identity, and Contradiction.

The three formal Laws of Thought are generally formulated thus:

(1) The Law of Identity. Whatever is is; or symbolically, A is A.

(2) The Law of Contradiction, or better, Non-Contradiction. Nothing can both be and not be; or symbolically, A is not not-A.

(3) The Law of Excluded Middle. Everything must either be, or not be; or symbolically, A is either B or not B.

Put in this bald way the Laws cannot be said to be very impressive. They are mere truisms, or mere tautologies. What do we learn from $A = A$, it may be asked? Obviously nothing. But if we did learn any particular truth from it, it would not be an adequate enunciation of a principle underlying all truth. In so far as a judgment or a reasoning conforms to the

Laws of Thought it becomes more and more obviously valid ; but it also becomes less and less valuable for the information it gives. Only purely symbolical reasoning *absolutely* conforms to them; other reasoning approaches this standard with varying degrees of perfection, from the loose inferences of everyday practical life, through the more accurate planes of law and medicine, and the still more accurate planes of physical science, until we come to the almost perfectly formal thought of mathematics, and the perfectly formal thought of Symbolic logic.

The principle of Identity proclaims that an object of thought must be regarded as identical with itself. A concept must remain unchanged while it is being manipulated ; and the same symbol must be regarded as standing for exactly the same denotation and connotation all through the process.

There is no need to dwell on this. But it is important to note that nearly every actual material judgment, that is, every judgment about an actual matter of thought as opposed to a mere symbolical judgment, to some extent infringes the principle. As Lotze, an illustrious German philosopher (died 1881), puts it, it is necessary to justify the categorical judgment "A *is* B." Formally speaking "A is A," and in so far as B is not A, A is not B. If I say a "dog is an animal," I am regarding as equivalent two very different objects of thought. A dog is an animal, but he is a great deal more ; he is a very specific animal, and I certainly cannot substitute animal for dog, and dog for animal,

without running into error. Only purely tautologous propositions absolutely comply with the principle; *e.g.*, "A dog is a dog," "John is John." But these convey no knowledge of any kind; and faint shades of difference are perceptible in subject and predicate when we use a tautologous proposition in such a way as to convey meaning. If I say "John is always John, you know," I mean something like this, that "John at the present time, however different he may appear, is the same John whose character is so well known to us." Different shades of meaning attach to the subject and predicate of "A pound is a pound," "A man's a man;" and it is just in this difference that the meaning, the material content of the proposition, lies.

The Law of Contradiction means that what is self-contradictory is unthinkable; or as Mill puts it, "the affirmation of any assertion, and the denial of its contradictory are logical equivalents." It, of course, does not mean that you must never say that "A is both B and not-B." The blotting paper in front of me is both white and not white; parts of it are white, and parts are black with ink, and even the parts I call white are not so white as to quite exclude black; and the blots are not *quite* black. The principle of Contradiction means that the *same* object of thought, the same indivisible thing, A, absolutely defined here and now, cannot both be B and not-B, when B and not-B are marked off with absolute logical precision. The more precise we are, the more will our judgments conform to this principle; but they will never absolutely

conform as long as we are thinking of actual objects of perception and conception, and not purely abstract ideas.

When we actually employ negative judgments we imply a certain sameness underlying the contradiction. If I say "A dog is not a wolf," I mean more than that the two are quite different; and the significance and value of the judgment mainly consists in the degree of similarity which underlies the difference. If I say "A dog is not a Gregorian chant," or (with the Clown in "Twelfth Night"), "The Myrmidons are no bottle-ale houses," my proposition has practically no significance because the contradiction is so absolute.

In actual use, then, a judgment rests on both principles. There is both identity and contradiction underlying it. But for logical purposes we must disregard the contradiction which underlies an affirmative judgment; and the identity which underlies a negative judgment. Otherwise precise, formal inference will not be possible. In the same way the mathematician knows that all actual concrete units differ amongst themselves, that no two yards of cloth, or pounds of sugar, are *absolutely* alike. But he knows that his results are strictly true only on the assumption that his units are absolutely the same and unalterable.

§ 3. Law of Excluded Middle.

The Law of Excluded Middle tells us that if we take any two objects of thought, A and B, whatever they may be, A either is B or is not B.

This, like the other laws, is only true in the abstract. It presupposes that A and B are absolutely defined; and it presupposes that not-B includes everything that is outside the class B, *i.e.*, that B and not-B are logical contradictions. (See Chap. II. § 9). It is clear that "A plane rectilineal figure is three-sided, or not three-sided," and here we are conscious of no difficulty. But if I say "Virtue is blue, or not blue," we feel a difficulty, because blue and not-blue are usually understood as limited contradictories, and the latter as well as the former ordinarily implies that the object is coloured. Reference is not made to things in general, but to things susceptible of colour. The *universe of discourse*, as it is called, is tacitly limited to coloured objects. The principle of Excluded Middle of course assumes that we are using "not blue" as the strict logical contradictory of "blue," as an *infinite* term applying to every object of thought which is not blue—to virtues and vices, political institutions, mathematical formulæ, forces of Nature, and so on, as well as to concrete objects of visual perception which are susceptible of colour. We at once frankly own that no thinkable meaning attaches to an infinite term, that it is quite unthinkable because it has an infinite denotation and next to no connotation at all. But no thinkable meaning attaches to an absolutely identical judgment, or an absolutely contradictory one. No actual thinkable judgments, as we have just said, absolutely conform to the requirements of the Laws of Identity and Contradiction.

§ 4. Laws of Homogeneity, Heterogeneity, and Continuity.

Over against the purely formal principles of Thought we may set three principles which are always found in actual thinking, and which are, in fact, a recognition of the truth that, in actual thought, the conditions of purely abstract thought are never entirely fulfilled.

These three regulative principles which refer to the content or material of thought may be stated as follows :—

(1) *Law of Homogeneity.* However different any two concepts may be, they can always be brought under some higher concept. That is, the most dissimilar things must be in some respects similar.

This is the complementary truth to the law of Contradiction. All actual negative judgments, as we have seen, imply some similarity. The principle of Homogeneity further tells us that generalization is always possible, and that we never arrive at a *summum genus* beyond which generalization can go no further. When we deal with purely formal concepts, with A's and B's, the law has no meaning. But in the actual world of perception, and in the world of conception, in so far as it is based on this outer world, we are constantly under obligation to it. (See Chap. XIV. § 1, below.)

(2) *Law of Heterogeneity.* Every concept contains other concepts under it. That is, the most similar things are in some respects dissimilar.

This is the complement to the principle of Identity,

as the principle of Homogeneity is to the principle of Contradiction. It implies that specialization is always possible; that two things or classes, as long as they are in any sense two, must have some difference, no matter how great their likeness. It tells us that we can never get a true proposition of the form "A = A," so long as we are dealing with actual objects of thought; and that an *infima species* is always impossible.

(3) *Law of Continuity.* No two co-ordinate objects of perception touch each other so closely but that we can conceive another or others intermediate. *In mundo non datur hiatus.*

This complements the Law of Excluded Middle. When one has perfect classes the Law of Excluded Middle is true. If we define *warm* water as water having a temperature of 60° Fahrenheit and upwards, then any water which falls an infinitesimal part of a degree below 60° is not warm, and any given glass of water whatever will be either warm or not warm. But in actual practice the matter is not so simple. Differences of temperature between 59° and 60° Fahrenheit would not be detectable by skin sensations. Very minute differences of temperature are not detectable at all. Suppose we have several good thermometers, and they severally give us the temperature of the water as 59·98°, 59·99°, 60·0°, 60·02°, is the water warm or not warm? Besides, the different portions of the fluid will vary in temperature, it will be cooler at the top than in the middle;

and the temperature at one moment will not be the temperature a few seconds later.

Modern science is always insisting on the principle of continuity. The older scientific men of the seventeenth and eighteenth centuries, dazzled by the brilliant achievements of the science of mathematics, were anxious to assimilate all other sciences as far as possible to the same type. But they forgot that mathematics deals with highly abstract concepts and not with concrete things. The same precision of definition and of division which is possible in geometry is quite impossible in biology. In the sciences of life one class merges into another, and definition is always provisional and tentative. And now generally we may say that in the world of experience the limits between phenomena are not drawn with a rigid line. The cases of the museum imply a discreteness which does not exist in actual experience. Nature cannot be pigeon-holed.

The same thing is true, though less markedly, of human thoughts or concepts. "Right" and "not right," "true" and "not true," are not determinable with the exactness that A and not-A are. Consequently when formal logic requires to answer Yes *or* No, we must often say neither Yes nor No, or else both of them.

§ 5. Principle of Sufficient Reason.

The principle of Sufficient Reason was first formulated by Leibnitz, and put by him on the level of the

principles of Identity and Contradiction. He states it thus: "Nothing can exist or be true unless there is a sufficient reason why it should exist thus and not otherwise, although these reasons are unknowable."

This has a double meaning. It refers to things and to judgments. It says that there must be a ground or reason for the existence of everything, and that there must be a ground or reason for the truth of every judgment. The former principle belongs to the realm of Nature, that is, of perception. It is a pre-requisite of any ordered experience, and therefore of any knowledge of Nature. It is generally known as the Law of Causation, and is discussed below in Chapter XIV. The second, or purely logical principle is not, to use Mansel's words, a law of thought, but only the statement that every act of thought must be governed by some law. It is not capable of purely abstract and symbolical expression, like the Laws of Identity and Contradiction, for it requires the introduction of a definite notion (Reason or Condition), which is a special object of thought, and not the representation of all objects whatever.

CHAPTER VIII.

IMMEDIATE INFERENCE.

§ 1. Inference.

By inference we mean the process by which we pass from a given proposition, or propositions, to a fresh proposition, which is true if the former be so. The relation between the new proposition (called the *conclusion*) and those from which we infer it (called the *premises*) must be such that the inference is recognized as valid by all normally constituted minds, or the logician will not accept it.

Logic, as we have seen, does not profess to describe the actual process of inference ; but it offers tests of valid inference, and prescribes certain forms into which the inference must be thrown if we wish to pass judgment on its validity. With this end in view, the logician recognizes two broadly different types of inference, Deduction and Induction (see Chap. I. § 4), and the former he divides into Immediate Inferences, where a conclusion is drawn from a single given proposition or premise, and Mediate Inference, in which two or more premises are necessary.

It is with those Immediate Inferences, called by

Welton *Eductions*, that we have to do in this chapter. From a single given proposition we deduce other propositions differing from it in subject, in predicate, or in both, whose truth is implied by the truth of the given proposition.

Eductive Inferences are of two principal kinds and two subordinate kinds: Obversion, Conversion, Contraposition, and Inversion.

§ 2. Obversion.

By obversion we arrive at a new judgment which has the same subject as the given proposition for its subject, but the contradictory of the old predicate for its predicate. The new proposition (called the *obverse*) is of different quality from the original (called the *obvertend*), but of the same quantity. It is of course true if the original proposition is true.

Rule for obversion. Instead of the predicate of the obvertend, write its contradictory; then change the quality of the proposition, but nothing else.

Thus:—"All S is P," because "No S is not-P;" "No S is P," because "All S is not-P;" "Some S is P," because "Some S is not not-P;" "Some S is not P," because "Some S is not-P." To take the last in detail. The obvertend is, "Some S is not P;" write "not-P" instead of "P," and "Some S is" instead of "Some S is not." We get "Some S is not-P."

Before attempting to deal with an actual proposition, we must make up our mind exactly what it

means; which is the subject and which the predicate. Thus, if we have to obvert "Only animals feed," we must note that this is an exclusive proposition and includes two propositions. It means that "No not-animals feed," and that "Some animals feed;" better written for our purpose, "No not-A are F," and "Some A are F" (see Chap. III. § 4). The former becomes on obversion "All not-A are not-F" (= "All things which are not animals are things which do not feed"); the latter becomes "Some A are not not-F" ("Some animals are not things which do not feed").

The student will notice, that while obversion in the case of universal propositions gives us results of some interest, it does not do so in the case of particulars. The opposition between the subalterns is so small that the inference is scarcely perceptible.

When we know that there is no possibility of mistake, we may often substitute for the strict contradictory of the predicate the contrary. For instance, if the obvertend be "All the members of the club are women," we get strictly the obverse, "No members of the club are not-women;" but as we know that only human beings are members of clubs, we may substitute "men" for "not-women," and say, "No members of the club are men."

What is sometimes called *material obversion* is not obversion at all. The process of thought by which from "Warmth is agreeable" we proceed to "Cold is disagreeable," is not—notwithstanding Dr. Bain's

authority—a deductive inference. It may be an induction, resting on fresh experience. It certainly cannot be brought under the definition of obversion given above.

§ 3. Conversion.

By *conversion* we mean that process of immediate inference by which, from a given proposition (called the *convertend*), we arrive at a new proposition (called the *converse*), which has for its subject the predicate of the original proposition, and *vice versâ*. This new proposition will be of the same quality as the original, but not necessarily of the same quantity.

The essential condition of valid conversion is, that no term in the converse is to be distributed unless it is already distributed in the convertend. Thus E and I propositions can be converted without any difficulty: "No S is P," becomes "No P is S," and "Some S is P," becomes "Some P is S." This is called *simple conversion*. In the case of A, however, as the predicate is not distributed (Chap. III. § 3) we cannot convert simply.

We cannot infer from "All S is P" that "All P is S." If all mushrooms are plants, it does not follow that all plants are mushrooms. We must take care that P preserves its undistributed character in the converse; so we get "Some P is S," which of course does not mean that *only* some P is S, but that some P at least is S (Chap. III. § 3). This kind of conversion, which

infers an I proposition from an A proposition, is called *conversio per accidens*, or *conversion by limitation.*

When an A proposition is known to have its predicate distributed, *e.g.*, "All men are rational animals," where the denotation of subject and predicate is the same, simple conversion may be applied. But this presupposes special information with regard to the subject matter over and above what is given in the proposition itself.

O propositions cannot be converted at all. For the subject of the convertend is undistributed; and if this is made the predicate of a negative proposition it must be distributed: "Some S is not P." would have to become "Some P is not [any] S." Now we know nothing about all S's, though we know something about some S's, and, therefore, we cannot make an assertion which involves a knowledge about all S's.

Beginners must be careful to remember that every change in the position of subject and predicate is not necessarily a logical conversion. We do not convert "Great is Diana of the Ephesians" by writing "Diana of the Ephesians is great," because this last form is the exact logical expression of the original proposition, of which the subject is "Diana of the Ephesians," and not "great." The true converse is, "One (or some) great being is Diana of the Ephesians."

§ 4. Contraposition.

By *contraposition* we mean a process of immediate inference by which, from a given proposition, we arrive at another which has for its subject the contradictory of the original predicate, and the original subject for its predicate.

The contrapositive is sometimes called the converse by negation. This name, however, should be avoided.

Rule for contraposition.—Obvert the given proposition and then convert the obverse.

Thus, "All S is P," becomes (1) "No S is not-P;" (2) "No not-P is S."

"No S is P," becomes (1) "All S is not-P;" (2) "Some not-P is S."

"Some S is not P," becomes (1) "Some S is not-P;" (2) "Some not-P is S."

An I proposition cannot be contraposited, because the first step, obversion, gives us an O proposition, which cannot be converted.

As an example for contraposition let us take the proposition, "All men pursue their own interest." This is an A proposition, and may be written, "All men are beings who pursue their own interest." The obverse of this is, "No men are beings who do not pursue their own interest;" and the converse of this obverse is, "No beings who do not pursue their own interest are men."

Here is another example: "Less than half the members are in favour of the Local Veto." The

logician can pay no attention to numerical statements, so this statement must be taken as equivalent to two propositions: "Some members are not in favour of the Local Veto," and (probably), "Some members are in favour of the Local Veto."

The former is an O proposition; its obverse will be, "Some members are not-favourers of the Local Veto;" while the converse of this last is "Some who do not favour the Local Veto are members." The I proposition, "Some members are in favour of the Local Veto," which is probably implied by the original statement can be obverted, and becomes "Some members are not persons not in favour of the Local Veto;" or, to be more concise, let us say, "Some members are not persons opposed to the Local Veto." This is an O proposition and cannot be converted.

§ 5. Inversion.

Inversion is that process of immediate inference in which, from a given proposition, we infer another proposition having for its subject the contradictory of the subject of the original proposition, and for its predicate the original predicate The quantity and quality of the new proposition, or *inverse*, will be the opposite of those of the original proposition, or *invertend*; in other words, an A proposition will have an O for its inverse, an E will have an I for its inverse. Only universals can have an inverse.

Rule for inversion.—Perform conversion and obver-

sion alternately, beginning with conversion in the case of an E proposition, and obversion in the case of an A proposition, until a proposition of the required form is obtained. Thus, "All S is P," becomes (1) "No S is not-P" (obverse); (2) "No not-P is S" (contrapositive); (3) "All not P is not-S" (obverted contrapositive); (4) "Some not-S is not-P" (converse of obverted contrapositive); (5) "Some not-S is not P" (obverse of converse of obverted contrapositive).

And "No S is P," becomes (1) "No P is S" (converse); (2) "All P is not-S" (obverted converse); (3) "Some not-S is P" (converse of obverted converse).

We see that only three steps are necessary in the case of an E propostion; while five are required to obtain the inverse of an A proposition.

If we reach an O proposition, before the inverse is obtained, and have to convert this, the process must come to an end. Thus I and O propositions yield no inverse.

N.B.—For Immediate Inferences by Opposition, see above, Chap. III. § 5.

CHAPTER IX.

THE SYLLOGISM.

§ 1. The Syllogism.

DEDUCTIVE mediate inference is of two kinds, categorical and conditional. The most important kind of mediate inference, in which only categorical propositions are employed, is the categorical syllogism. Other forms exist, but these usually take the form of a group of syllogisms (*e.g.*, *sorites*), or imitate the general arrangement of the syllogism.

The syllogism consists of three propositions containing three terms. The conclusion is usually placed last, and the propositions on which it rests, called the *premises*, are placed above it. The predicate of the conclusion is called the *major term;* the subject of the conclusion is called the *minor term.* In one of the premises properly placed first, appear, the major term, and a third term called the *middle term;* in the other premise, placed second, appear the minor term and the middle term. Thus using S, P, and M for the minor, major, and middle terms, and placing the premises and terms in the most usual order, we should have, if each of the propositions were an A proposition,

All M is P,
All S is M,
∴ All S is P.

Or as a concrete example:

All mammals are warm-blooded,
All whales are mammals,
∴ All whales are warm-blooded.

We can now see why "warm-blooded" is called the major term, and "whales" the minor term. The former has the widest denotation; it includes the class mammals, which includes the class "whales." "Whales" then has the least denotation of all three.

The premises are called, after the term they contain, the *major premise* and the *minor premise* respectively.

§ 2. Axioms of the Syllogism.

Aristotle regarded the following statement, called the *Dictum de omni et nullo*, as the fundamental principle underlying all mediate reasoning; the simplest and most self-evident way in which the process could be justified.

"Whatever is predicated—affirmatively or negatively—of a distributed term, may be predicated in like manner of anything which is included under it." If we can make an assertion about all the class M, we may make the same assertion about any thing which

belongs to this class M. If "Every M is P," and "S is M," then "S is P."

This axiom may be regarded as a special application of the first two laws of thought; what is true of A is true of A; what is not true of A is not true of A. It is applicable to the form of the syllogism just given, though not to all forms of it.

Alternative axioms have been suggested, some of which are restricted, like Aristotle's, to one particular form of syllogism, *e.g.*, the "Nota notæ est nota rei ipsius. Repugnans notæ, repugnat rei ipsi," which Mill renders thus: "Whatever is a mark of any mark, is a mark of that which this last is a mark of."

This form lays most stress on the relations to connotation of the terms, as the others do in the relations to denotation.

Others are wider, such as Whately's:

"(1) If two terms agree with one and the same third, they agree with each other.

"(2) If one term agrees and another disagrees with the same third, these two disagree with each other."

§ 3. Rules of the Syllogism.

From these axioms the rules of the syllogism can be deduced.

The rules are as follows:

(1) A syllogism must have three, and only three, terms.

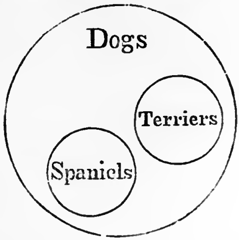
(2) A syllogism must have three, and only three, propositions.

(3) The middle term must be distributed once at least, and must not be ambiguous.
(4) No term must be distributed in the conclusion which is not distributed in its premise.
(5) From two negative premises nothing can be inferred.
(6) If one premise be negative, the conclusion must be negative ; and *vice versâ*, to prove a negative conclusion one of the premises must be negative.

The *dictum* clearly provides for three terms and three propositions. It provides for the distribution of the middle term, and that in the major premise ; for it tells us that what is predicated of a distributed term (the middle term) may be predicated of everything that can be brought into the class denoted by this term. The M must be distributed in the major premise, which must make an assertion about all and every M. But reflection will show, that whether distributed there or not, it must at any rate be distributed once. For otherwise there are practically two middle terms, *i.e.*, no middle term at all. If our premises are :

"Some dogs are spaniels,
All terriers are dogs,"

we are referring to two different sets of dogs ; there is no overlapping of the spaniels and terriers, because both terms refer to a different subdivision of the middle term. A diagram will make this plain.



Where the middle term is distributed, we know something about *all* the things contained under it, so that if the minor term is declared to belong to this class, we know something about the minor term. For the same reason the middle term must not be ambiguous ; if it is, it ceases to be a true middle term, and we have four terms in our syllogism (called the fallacy of *quaternio terminorum*).

Rule 4 warns us against distributing the major term in the conclusion if it has not been distributed in its premise (called *illicit process of the major term*, or shortly *illicit major*), and against distributing the minor term in the conclusion if it has not been distributed in its premise (*illicit process of the minor term*, or *illicit minor*). The second fallacy is directly forbidden by the words of the *dictum*, "included under it." The former is provided against indirectly, since in the arrangement of terms given, called the *first figure*, to which alone Aristotle's *dictum* applies, the major term will only be distributed in the conclusion, if the conclusion is negative, and this can only be when the major premise is negative, according to the dictum, for the

conclusion must predicate "in like manner" (*i.e.*, affirmatively or negatively) as the major premise. This gives us also the sixth rule, while the fifth is secured by insuring an affirmative minor. We can only *include* the minor term under the middle term in an affirmative premise.

By generalizing we see that the rules are all valid and important, whatever may be the order in which the terms stand.

The essential thing is to secure the overlapping of the classes denoted by the terms. And this can best be done by observing the rules. Obviously, without the first three rules, we have no syllogism at all.

The fourth rule forbids our asserting what we know to be true of some members of a class (but do not know to be true of all members of the class) to be true of all. In illicit minor the fallacy is usually barefaced. Thus:

All M is P,	No M is P,
Some S is M,	Some S is M,
∴ *All* S is P.	∴ *No* S is P.

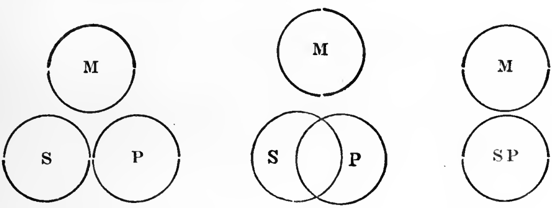
But in illicit major the mistake is often more concealed. Thus, at first sight, the following looks a valid syllogism:

All M is P,
No S is M,
∴ No S is P.

But on examination we see that P has been distributed in the conclusion (predicate of a negative proposition), while it is undistributed in its premise.

We first asserted that all M is (some) P, and we now proceed to make an assertion which implies a knowledge of *all* P's. All we know is that S is excluded from some P's, and we are asserting that S is excluded from the whole class.

Rule 5 deserves a word or two. Two negative premises are not permitted by the *dictum* of Aristotle, because that provides for an affirmative minor. And we can see that to exclude S and M from each other, and P and M from each other, tells us nothing about the relations of S and P. They may be entirely apart, as at (I), or they may partially overlap, as at (II), or they may entirely overlap, as at (III):



(I) (II) (III)

In some cases the rule may appear broken when it is not. Thus:

None but Radicals are members of the club,
The Robinsons are not Radicals,
∴ The Robinsons are not members of the club.

Here, however, the two premises may be written as follows:

No not-Radicals are members,
The Robinsons are not-Radicals,

when "not-Radical" (a negative term) becomes the middle term, the minor premise having been obverted.

Two further rules are sometimes given, viz.:

(7) From two particular premises nothing can be inferred.

(8) If one premise be particular, the conclusion must be particular.

These are corollaries from the first six rules, and require some study of the syllogism before they can be deduced. They should not be used in solving problems on the syllogism in pass examinations.

§ 4. The Valid Moods.

If we write out all the combinations of A, E, I, O, two together, we shall have the following sixteen arrangements:

A A	E A	I A	O A
A E	E E	I E	O E
A I	E I	I I	O I
A O	E O	I O	O O

These are the pairs of premises which are *primâ facie* possible; each pair is supposed to contain a middle term. We have to see that no rule of the syllogism is broken. Obviously E E, E O, O E, and O O break Rule 5, and therefore cannot stand. I I will have an undistributed middle, since the two premises contain not one dis-

tributed term. I E and I O will break Rule 4, for the conclusion must be negative, and the predicate of the conclusion, or major term, must therefore be distributed; it is, however, undistributed in its premise, since in I both terms are undistributed. This leaves us only nine combinations *prima facie* allowable, A A, A E, A I, A O, E A, E I, I A, O A, O I.

Each of these pairs of premises might have for its conclusion any one of the four forms of proposition. Thus: A A A, A A E, A A I, A A O; A E A, A E E, A E I, A E O, etc. These groups of three propositions (which are, of course, assumed to have the three terms, major, minor, and middle) are called *moods* of the syllogism (Latin, *modi,* shapes or arrangements). We have thirty-six of them to examine. But many of these arrangements obviously break Rule 5 or 6; *e.g.*, A A E, A A O, A E A, A E I. Some others break Rule 4; *e.g.*, those with a universal conclusion and a particular affirmative minor premise. After rejecting all those moods which necessarily break Rules 4, 5, and 6, we are finally left with only eleven arrangements, A A A, A A I, A E E, A E O, A I I, A O O, E A E, E A O, E I O, I A I, O A O.

Each of these is a *valid mood* of the syllogism.

§ 5. Figures of the Syllogism.

The two premises of the syllogism each contain the middle term, and the first one the major term, the second the minor term. Obviously four arrangements

are possible. Denoting the middle term by M, the major by P, and the minor by S, we may have:

M P	P M	M P	P M
S M	S M	M S	M S
(I)	(II)	(III)	(IV)

These are called the four *figures* of the syllogism.

Figure, then, has nothing to do with the kind of propositions employed, but simply indicates the position of the middle term in the premises.

Each of the eleven valid moods is valid in one at least of the four figures, but not necessarily in more than one. To see in what figures any particular combination is permissible, we have to examine it in each of the four. Thus A A A takes the following four forms:

All M is P,	All P is M,
All S is M,	All S is M,
∴ All S is P.	∴ All S is P.
(I)	(II)
All M is P,	All P is M,
All M is S,	All M is S,
∴ All S is P.	∴ All S is P.
(III)	(IV)

The only rules which may have been broken are the third and fourth; infraction of the others has already been guarded against. A A A, we can see, is only allowable in Fig. I.; in Fig. II. it breaks Rule 3; in Figs. III. and IV. it breaks Rule 4 (illicit minor).

After trying each of the eleven moods in all four figures we find that—

In Fig. I., A A A, E A E, A I I, E I O (A A I) and (E A O) are valid.

In Fig. II., E A E, A E E, E I O, A O O (E A O), and (A E O).

In Fig. III., A A I, I A I, A I I, E A O, O A O, E I O.

In Fig. IV., A A I, A E E, I A I, E A O, E I O (A E O).

The moods inclosed in brackets are called *subaltern moods;* they have a weaker conclusion than one is permitted to draw. It is clear, that whenever a universal conclusion is allowable, a particular conclusion may be drawn. There are no subaltern moods in Fig. III., because no universal conclusion is permissible in that figure.

The student should exercise himself in drawing up symbolic syllogisms in the different moods and figures, and then writing concrete syllogisms of the same shape. Let us take A I I, in Fig. III., for instance. The middle term is subject in both premises, so we shall have:

All M is P,
Some M is S,
∴ Some S is P.

The following concrete syllogism is in the above form:

All abstract principles are difficult to grasp thoroughly,

Some abstract principles are implied in the simplest reasonings,
∴ Some things implied in the simplest reasonings are difficult to grasp thoroughly.

§ 6. The Special Rules and Canons.

A little examination shows that there is considerable difference between the figures. The first figure is by far the most natural and the most valuable. It proves conclusions of all four kinds, A, E, I, O, which no other figure does. It complies easily with Aristotle's *dictum*, and hence was called by him the perfect figure.

On the other hand, the fourth figure was not recognized by Aristotle at all; it was added to the other three by the famous physician and philosopher, Galen, in the second century after Christ, and it is still sometimes called the Galenian figure. It is rejected by many logicians as awkward, and practically valueless; but this is no reason why it should not have its proper place in the theory of the syllogism as completing the scheme of possible arrangements of the three terms.

The second figure proves only negative conclusions. Why this is so can be seen after a moment's reflection. The middle term is the predicate of both premises; therefore if both premises are affirmative it will not be distributed. But if one of the premises is negative, the conclusion must be negative.

The third figure, as we have seen, can only prove

particulars. The reason of this is as follows:—The middle term is the subject of both premises. If both premises be affirmative, the conclusion must obviously be an I proposition. If one be negative, the conclusion must be negative ; and this will involve P being distributed in the conclusion and in its premise; therefore the major premise will be negative, and the minor, affirmative. But if the minor is affirmative, S is undistributed, and the conclusion will be an O proposition. Thus, in any case, the conclusion will be a particular proposition. The third figure "is frequently useful when we wish to take objection to a universal proposition laid down by an opponent, by establishing an instance in which such universal proposition does not hold good. It is the natural figure when the middle term is a singular term, especially if the other terms are general" (Keynes).

Lambert, a famous German writer on physics and on logic, who died in 1777, says, "The first figure is suited to the discovery or proof of the properties of a thing ; the second to the discovery or proof of the distinction between things ; the third to the discovery or proof of instances and exceptions; the fourth to the discovery or exclusion of the different species of a genus."

Lambert is usually credited with having formulated *dicta* for the other figures corresponding to Aristotle's *dictum de omni et nullo* which applies to the first. These canons for the second and third figures run thus:

Dictum de diverso (Fig. II.). "If one term is contained in, and another excluded from, a third term, they are mutually excluded." Or better, as given by Mansel, "If a certain attribute can be predicated, affirmatively or negatively, of every member of a class, any subject of which it cannot be so predicated, does not belong to the class."

Dictum de exemplo (Fig. III.). "Two terms, which contain a common part, partly agree, or if one contains a part which the other does not, they partly differ." Or better, as expressed by Mr. Welton, "If anything which is stated to belong to a certain class is affirmed to possess, or to be devoid of, certain attributes, then those attributes may be predicated in like manner of some member of the class."

The canon for the fourth figure (*Dictum de reciproco*) is not expressed by Lambert in the same general form; and logicians differ considerably in their statement of it. Mr. Welton gives the following:—"Whatever has a predicate affirmed, or universally denied, of it, may itself be predicated particularly and with like quality, of anything which is affirmed of that predicate; and whatever has a predicate universally affirmed of it may itself be universally denied of any thing which is universally denied of that predicate."

Special rules may be deduced for each of the four figures from its special canon. Those of the first are:

(1) The major premise must be universal.

(2) The minor premise must be affirmative.

Those of the second figure are:

(1) The major premise must be universal.
(2) One of the premises must be negative.

Those of the third figure are:

(1) The minor premise must be affirmative.
(2) The conclusion must be particular.

Some of these have already been explained, and the student should work out all for himself. There is no better exercise in easy abstract reasoning.

§ 7. Reduction.

Aristotle regarded the first figure as of peculiar validity and cogency because it directly complied with the *dictum*, which he considered the principle underlying all mediate inference. Hence it became important to show that syllogism in the other or *imperfect figures* could be expressed in the first figure, and the same conclusion obtained in that figure. This process he called *Reduction.*

The opinion on which the need for Reduction rested no longer exists; but it is interesting to show that all the different forms of the syllogism are equivalent and consistent, though not for all purposes equally convenient.

By the processes of obversion and conversion, together with the change of premises, any valid syllogism for Figs. II., III., and IV. can be expressed in a corresponding mood of Fig. I. Or speaking generally, a

valid syllogism can be expressed in any of the four figures, though of course not in the *same* mood.

The rules for reducing the imperfect figures are contained in some mnemonic verses, which also record the valid moods of each figure. In the form usually given these run:

> "Barbara, Celarent, Darii, Fĕrioque, *prioris;*
> Cesare, Camestres, Festino, Baroko, *secundæ* ;
> *Tertia* Darapti, Disamis, Dātisi, Felapton
> Bokardo, Ferison, *habet;* *Quarta insuper addit*
> Bramantip, Camenes, Dĭmaris, Fesapo, Fresison."

The words in italics are real Latin words which tell us that the words in the first line belong to the first figure, those of the second line to the second, and so on. The other words are, of course, merely arbitrary. The vowels of each give us the mood: thus Barbara stands for A A A; Bokardo for O A O. The following consonants are significant of the rules for conversion, viz., *s*, *p*, *m*, and they therefore do not appear in the moods of Fig. I. The other consonants are mainly euphonic; but we note that any mood in the so-called *imperfect figures* which begins with B will reduce to the mood in the first figure which begins with that letter, and so with C, D, and F.

s signifies simple conversion.

p signifies conversion *per accidens*

m signifies change of the order of premises (mutation).

In each case the consonant refers to the premise it follows.

Let us take as our first example Festino. All we have to do is to convert simply the major premise. Thus:

$$\left.\begin{array}{l} \text{No P is M,} \\ \text{Some S is M,} \\ \therefore \text{Some S is not P,} \end{array}\right\} \text{ becomes } \left\{\begin{array}{l} \text{No M is P,} \\ \text{Some S is M,} \\ \therefore \text{Some S is not P.} \end{array}\right.$$

Here we have a syllogism in Ferio. A more elaborate case is that of Disamis, where we have to convert simply both the major premise and conclusion, and transpose the premises. Thus:

$$\left.\begin{array}{l} \text{Some M is P,} \\ \text{All M is S,} \\ \therefore \text{Some S is P,} \end{array}\right\} \text{ becomes } \left\{\begin{array}{l} \text{All M is S,} \\ \text{Some P is M,} \\ \therefore \text{Some P is S.} \end{array}\right.$$

The reasoning is now entirely in Fig. I. (Darii); but we shall have to convert the conclusion again if we want to get it in exactly the same form as in the given mood.

The Aristotelian logicians did not employ the process of obversion for reduction, and it was found necessary to reduce Baroko and Bokardo by a special method. The reduction which we have described is called *direct* or *ostensive reduction.* Baroko and Bokardo were brought under Fig. I. by what is called *Indirect Reduction,* or *reductio ad impossibile.* This is analogous to the indirect proof which Euclid so often uses, known as the *reductio ad absurdum.* If you dispute the conclusion in Baroko or Bokardo, then I will show you—using only the first figure—that you are

driven to contradict one of your original premises. Thus, in the case of Bokardo :

Some M is not P,
All M is S,
∴ Some S is not P.

If the O conclusion is wrong, then the corresponding A proposition must be true ; if "Some S is not P" be false, then "All S is P." Substitute this for the major premise, and we get

All S is P,
All M is S.

So we draw, according to Barbara in Fig. I. (whence Bokardo begins with B), whose validity is unquestioned, the conclusion

All M is P.

But this directly contradicts our original minor premise. Thus we see that we cannot contradict the original conclusion without contradicting one of the original premises.

If we use obversion we can reduce Baroko and Bokardo directly, but not to Barbara. Baroko, by contrapositing the major premise and obverting the minor, becomes Ferio. Thus :

$$\left.\begin{array}{l}\text{All P is M,}\\ \text{Some S is not M,}\\ \therefore\ \text{Some S is not P,}\end{array}\right\}\text{ becomes }\left\{\begin{array}{l}\text{No not-M is P,}\\ \text{Some S is not-M,}\\ \therefore\ \text{Some S is not P.}\end{array}\right.$$

Bokardo is somewhat more difficult to deal with.

But if we take the contrapositive of the major premise, change the order of the premises, and write the converse of the obverse of the conclusion, we get a syllogism in Darii. Thus:

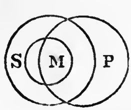
Some M is not P, All M is S, ∴ Some S is not P,	becomes	All M is S, Some not-P is M, ∴ Some not-P is S.

This last conclusion can be shown to be strictly equivalent to the original conclusion, for it can be reduced back to it by first converting and then obverting.

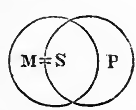
§ 8. Diagrammatic Representation of Syllogisms.

The student should practise reduction, using both symbolic and concrete syllogisms. He should also try to represent syllogisms by employing circles to represent terms, as on p. 97, above. These circles were frequently employed by a German writer on mathematics and logic named Euler (died 1783), and are often called *Eulerian diagrams.*

Bokardo may be represented thus—



; or thus—



M lies partly outside P, and M lies wholly within S, or else the two coincide. Therefore some S is in any case outside of P, namely, the S which is included in that part of M which is outside P.

This example already shows us one of the disadvantages of the method. Several diagrams may have to be used in order to adequately represent all the possibilities of the proposition. Here we have used two diagrams, because "All M is S" may mean "All M is a part of S," or "All M is all S." But the first premise is equally ambiguous, and really requires two diagrams to represent it.

"Some M" may mean "Some only (less than all) M;" or "All M," and we do not know which.

Consequently we should want four diagrams to represent the premises. Besides the two first given, we shall require these:

But notwithstanding this theoretical objection most students find considerable help in the use of Euler's diagrams, which often serve to illustrate and make clear the reasoning which they do not adequately represent.

CHAPTER X.

CRITICISM OF THE SYLLOGISM. COMPOUND AND IRREGULAR SYLLOGISMS.

§ 1. The Utility of the Syllogism.

THE syllogism is not put forward by modern logicians as a statement of the way in which men do actually reason, or as the way in which men must reason if they wish to reason validly. But it is put forward as a convenient method of testing mediate deductive inferences. We may adopt the words of Mr. Spencer: "The process of thought which the syllogism seeks to describe is not that by which the inference is reached, but that by which it is justified; and in its totality is not gone through at all unless the need for justification is suggested." In thought the conclusion comes first, and the proofs follow.

Most conclusions are presented to us first as an unproved theorem, a hypothesis. The mind usually reaches this by a process not fully conscious, and not representable by any arrangement of propositions. In most cases loose analogies, that is, resemblances more or less superficial, guide us to the theorems which we have to prove. When we want to prove our theorem

we have to look about for reasons, and the process becomes fully conscious.

The value of the syllogism, as Mill says, arises from its furnishing us with a mode in which our reasonings "may always be represented, and which is admirably calculated, if they are inconclusive, to bring their inconclusiveness to light" ("Logic," II. iii. 5).

§ 2. Mill's Attack on the Syllogism.

Mill denies that syllogism is really a process of reasoning ; it is, he says, merely a process of interpretation. He maintains that all reasoning is from particulars to particulars, when not an inference from testimony. "General propositions are merely registers of such inferences already made, and short formulæ for making more ; the major premise of a syllogism, consequently, is a formula of this description ; and the conclusion is not an inference drawn *from* the formula, but an inference drawn *according* to the formula ; the real logical antecedent being the particular facts from which the general proposition was collected by induction. . . . According to the indications of the record, we draw our conclusion : which is, to all intents and purposes, a conclusion from the forgotten facts. In this it is essential that we should read the record correctly: and the rules of the syllogism are a set of precautions to insure our doing so."

Mill's main error here is that he overlooks the fact that not only is the preliminary and, so to speak,

instinctive leap from the known to the unknown, from the observed cases in which fire burns, to the belief that any fire will burn, an inference; but that the process of logical proof is itself an inference. The one is an unjustified inference, the other a justified inference made according to rule, whose validity is therefore guaranteed. For what is an inference? We have seen (Chap. I. § 1), above, that an inference is a process of thought by which we proceed from a given proposition, or propositions, to a new proposition different from the former in form or matter, but so connected with the given proposition, or propositions, that if they be true the new proposition must be true. Now such an inference undoubtedly takes place in the syllogism. Even in ordinary examples, such as those taken by Mill, the conclusion is a new proposition differing both in form and matter from the major premise. And this would be clearer if Mill had taken less trite instances than "All men are mortal, Socrates is a man," etc. In the following syllogism there is unquestionable inference, the conclusion is obviously not guaranteed by a previous process of inference which led to the major premise—because there was no such process.

> All offenders against this law are liable to a year's imprisonment;
>
> The prisoner at the bar has offended against this law,
>
> ∴ The prisoner at the bar is liable to a year's imprisonment.

This is an example of what may be called the authoritative or judicial syllogism. Mill seems to have had a syllogism of this sort in his mind; but he tries to explain it by saying that "the object of the inquiry is to make out the . . . legislators' intention through the indication given by their words . . . . The operation is not a process of inference, but a process of interpretation" ("Logic," II. iii. 4). There is, however, no contradiction between interpretation and inference. Nearly all interpretation is inference. The application of the general law to the particular case of the prisoner is as much inference as anything can be. There is a new case, never before thought of; and a predicate which does not seem to apply to it is shown to do so, by means of the minor premise.

Mill further urges that if the syllogism be an inference there is a *petitio principii* in the conclusion. The fallacy of *petitio principii*, or begging the question (see Chap. XXI. below), consists in pretending to prove a given proposition by means of a proposition which can only be true if the conclusion is true. Mill insists that Socrates is mortal is assumed in the major premise, "All men are mortal." If Socrates is not mortal, the major premise is untrue. To this argument several answers are possible.

In the first place, the case of the authoritative syllogism is not open to this objection. Here at least there is no *petitio principii*, since the truth of the major premise is unaffected by the case of the prisoner at the bar.

Secondly, in ordinary syllogism, the major premise of which is the result of induction, it is clear that the fallacy does not lie in the syllogism, but in the preliminary induction. The deductive logician takes his premise from the inductive logician, and draws a conclusion from it. If the major premise is not warranted by the facts actually observed that is the fault of the inductive process which pretends to a certainty which it does not actually possess, not the fault of the deductive process which follows.

Again, whatever be the warrant on which the inductive major premise rests, "so long as in point of fact we *do* assert the major premise without first believing the conclusion, so long will the latter be an inference from the former" (Balfour, "Philosophic Doubt").

And, finally, even if we allow that the major premise is only true on condition that the conclusion is true, the same thing may be said of the solution of an algebraical equation. The equation

$$\frac{x}{a} - b = \frac{c}{d} - x$$

is only true on condition that

$$x = \frac{a\,(bd + c)}{d\,(a + 1)}.$$

Yet there is an undoubted process of inference in passing from the former to the latter statement, unless we are to entirely revise the meaning of the word

inference ; and there is nothing which can properly be called a *petitio principii* unless we are to regard every mathematical reasoning as a sham.

§ 3. Quasi-Syllogistic Arguments.

There are certain arguments which imitate to some extent the syllogistic form, and which are equally convincing ; but which are not true syllogisms.

The following is an instance :

B is taller than C,
A is taller than B,
∴ A is taller than C.

There are here, however, four terms, A, taller than C, B, and taller than B. The relation between A, B and C is a different kind of relation from the purely logical relation between subjects and predicates of propositions, and the relation cannot be expressed in a syllogism unless the material ground of the inference is expressed in one of the premises. We may say with Mansel :

> "Whatever is greater than a greater than C is greater than C,
> A is greater than a greater than C,
> ∴ A is greater than C ; "

and this frankly transfers the material ground on which the inference is based to the major premise, leaving only the logical inference to follow.

The argument *a fortiori* sometimes used by Euclid, which concludes, "much more then is A greater than C," is a special case of this kind of inference, which is a kind of reasoning not purely formal, but resting on certain very simple and immediately perceived *relations of degree.* Perhaps we may call such reasonings *quasi-syllogisms of degree.*

Another interesting kind of *quasi-syllogisms* are those which rest on the simpler and immediately perceived relations of space and time, *e.g.*

(1) Y lies to the east of Z,
X lies to the south of Y,
∴ X lies to the south-east of Z.

(2) John's death happened after Mary's marriage,
Kate's birth happened at the time of John's death,
∴ Kate's birth happened after Mary's marriage.

These are just as easily followed as syllogisms; but they are not due to pure or abstract thought, they rest on the perception of certain material truths resting on experience, which must be embodied in the major premise if the inferences are to be represented syllogistically.

Some attempt has been made to make out a "logic of relatives," but such a logic is unnecessary, and would be exceedingly cumbrous, and practically useless.

§ 4. Numerical Syllogisms. Ultra-total distribution.

Among other forms of a syllogistic character in which considerations which are not purely logical are introduced, we must include the so-called *numerically definite syllogism*. Here exact numbers are given, and the inference depends entirely on arithmetical relations and not on logical ones. Thus, if we know that out of one hundred instances there are seventy which are X, and fifty which are Y, at least twenty will be both X and Y. It is obvious that this kind of reasoning is not, in any proper sense of the term, syllogistic.

A form of reasoning which has more pretensions to the name syllogism than this, has been recognized by Hamilton and other logicians. Hamilton says that the rule of logicians that the middle term should be once at least distributed is unnecessary. For it is sufficient if, in both the premises together, its distribution be more than its quantity as a whole (ultra-total distribution). Therefore a *major part* (a *more* or *most*) in one premise and a *half* in the other are sufficient to make it effective. We may express a reasoning depending on this truth thus:

Most M is P,
Most (or at least half) M is S,
∴ Some S is P.

From purely numerical considerations we can see that

some cases of M will be both S and P, or in other words, that there will be cases of S which are P.

Most of the jury are named Brown,
Most (or at least half) of the jury are red-haired,
∴ Some red-haired men are named Brown.

Here we have a mood which the logician describes as I I I, and which is, from the point of view of Aristotelian logic, invalid. There is not a single distributed term. But it is valid, notwithstanding, because care has been taken to see that the denotation of subject and predicate overlap (cf. Chap. VIII. § 3). This, however, has been done by introducing numerical considerations (half, and more than half) which belong to mathematics and not to Logic.

§ 5. Enthymemes.

The word *enthymeme* is now used to signify a syllogism imperfectly expressed by the omission of one of the three propositions. If the major premise is omitted the enthymeme is said to be *of the first order;* if the minor premise is omitted, it is *of the second order;* if the conclusion, *of the third order.*

Mill's time-honoured example of the syllogism,

"All men are mortal,
Socrates is a man,
∴ Socrates is mortal,"

may be expressed in all three forms, thus:

(1) Socrates is a man, therefore he is mortal. (Or, Socrates is mortal, because he is a man).

(2) All men are mortal, therefore Socrates is mortal. (Or, Socrates is mortal, because all men are mortal.)

(3) All men are mortal, and Socrates is a man.

Such abridged expression is very convenient, and is more usual for ordinary argument than the full and technical expression, which would usually seem pedantic and affected, as well as cumbrous. In an argument of this sort one of the premises is usually much more obvious than the other. If someone says, "My tea is too sweet," it is only necessary to say, "I put three lumps of sugar in it," without enunciating the truth that "three lumps of sugar in a cup of tea make it very sweet." Even if someone asks, "Why is this leaf green?" it is commonly quite sufficient to say, "It contains chlorophyll," without adding the formal statement of the major, that those parts of plants which contain chlorophyll are green. This may not have been previously known, but it is certainly implied.

Aristotle used the term *enthymeme* to signify two different things, viz., either a syllogism of a probable character, resting on a proposition not universally, though generally, true (called a *plurative* proposition); or a syllogism which has for one of its premises a singular proposition. The former is not formally valid, but is very serviceable in practical life. To this class, indeed, belong most of the arguments which we require to use

in daily affairs. "Most fevers are catching," "Few errand boys can be trusted with a message," will give us conclusions which are not formally valid, since the middle term will be undistributed, but which will be probable enough to guide our conduct. *E.g.:*

Most fevers are catching,
This is a fever,
∴ This is (probably) catching.

The other class of syllogisms which Aristotle calls *enthymemes* are important only from the point of view of what may be called the archæology of Logic. The student may be referred to Mansel's edition of Aldrich's "Logic" (Appendix F), or to Mr. Welton's account ("Logic," pp. 467 *sq.*).

The word *enthymeme* is not directly derived from ἐν θυμῷ, as some writers have asserted, in the sense that one of the premises is kept "in the mind" and unexpressed. Indirectly it comes from ἐν and θυμός; but, as Mansel points out, the word ἐνθύμημα had been used by writers before Aristotle, in a non-technical sense, to mean a thought or suggestion.

§ 6. Prosyllogisms.

In most cases even an argument cannot be fully expressed in a single syllogism. The premises are not ordinarily self-evident, and some proof is required of one or both of them. A syllogism used to prove one of the premises of another syllogism is called a

prosyllogism; while the second syllogism, one of whose premises is thus proved, is called an *episyllogism.* Such a combination is a *train of reasoning* or *polysyllogism.*

Most of such chains of reasoning will be imperfectly expressed. Some of the prosyllogisms will be enthymemes. Such an abridged train of reasoning is called an *epicheirema.*

The following will serve as an example:

> All Socialistic schemes are at present impracticable, because they demand a higher moral development than the average man has yet attained,
> This is a Socialistic scheme, because it involves the ownership of all the capital and land by the State,
> ∴ This scheme is at present impracticable.

Each of these premises has a prosyllogism imperfectly expressed. The enthymeme of the second order, which forms the first prosyllogism, would be fully expressed thus:

> All schemes which demand a higher moral development than the average man has yet attained, are impracticable,
> All Socialistic schemes are schemes of this sort,
> ∴ All Socialistic schemes are impracticable.

The student can easily express the second prosyllogism for himself.

The name *epicheirema* has been therefore given to a chain of reasoning in which one or both of the premises is given as the conclusion of an imperfectly expressed syllogism or enthymeme. Thus the following is an epicheirema:

All M is P, because M is X,
All S is M, because S is Y,
∴ All S is P.

§ 7. Sorites.

A sorites may be regarded as a chain of syllogisms of which the conclusions, except the last, are suppressed. It is a chain of prosyllogisms all of which are enthymemes, while even the final episyllogism is an enthymeme.

There are two forms of sorites, one called the Aristotelian, and the other the Goclenian—from a German logician named Goclenius, who taught at Marburg in the seventeenth century. The Aristotelian may be expressed symbolically thus:

All A is B,
All B is C,
All C is D,
All D is E,
All E is F,
∴ All A is F.

Here the first premise contains the minor term of the final syllogism. The sorites may be broken up

into a series of syllogisms with the second, third, and subsequent propositions for major premises:

(1) All B is C,
All A is B,
∴ *All A is C.*
(2) All C is D,
All A is C,
∴ *All A is D.*
(3) All D is E,
All A is D,
∴ *All A is E.*
(4) All E is F,
All A is E,
∴ All A is F.

The suppressed conclusion of each prosyllogism becomes the implied *minor* premise of the succeeding syllogism. In the above syllogism the propositions which do not appear in the sorites are printed in italics.

The Goclenian sorites may be expressed thus:

All A is B,
All C is A,
All D is C,
All E is D,
All F is E,
∴ All F is B.

Here the first proposition contains the major term or predicate of the conclusion; the suppressed con-

clusion of each prosyllogism serves as *major* premise to the next syllogism:

(1) All A is B,
All C is A,
∴ *All C is B.*
(2) *All C is B,*
All D is C,
∴ *All D is B.*
(3) *All D is B,*
All E is D,
∴ *All E is B.*
(4) *All E is B,*
All F is E,
∴ All F is B.

The Aristotelian sorites consists of major premises, except the first (which is a minor), and the last (which is the conclusion). The Goclenian consists of minor premises, except the first (which is a major premise), and the last (which is the conclusion).

A sorites is at least as immediately convincing as the chain of syllogisms into which it can be decomposed. It is, in point of fact, a simpler form than that into which our analysis resolves it. The analysis is of no use, except to show that the sorites *can* be expressed syllogistically.

CHAPTER XI.

CONDITIONAL REASONINGS.

§ 1. Hypothetical Syllogism.

THE term hypothetical syllogism has been commonly applied to a form of mediate reasoning which has two premises; the first (usually called the *major* premise), a hypothetical proposition, and the second (the *minor*), a categorical one, while the conclusion is also categorical. The term hypothetical is here used in the sense given above (Chap. IV. § 1), and includes those which have been called material hypotheticals (Dr. Keynes' *conditionals*), as well as true or formal hypotheticals.

The only rule which need be remembered is this: "The antecedent must be affirmed, or the consequent denied."

The hypothetical syllogism which affirms the antecedent gives us an affirmative conclusion; that which denies the consequent gives us a negative conclusion. The former is therefore called the *modus ponens*, or affirming mood; the latter, the *modus tollens*, or denying mood.

We may express them symbolically, thus:

(1) *Modus ponens.*

If A is B, C is D,
But A is B,
∴ C is D.

(2) *Modus tollens.*

If A is B, C is D,
But C is not D,
∴ A is not B.

The reason why *denying the antecedent* and *affirming the consequent* cannot give us a valid ground of inference is obvious. There may be other conditions under which C may be D, as well as the one given, viz., that A is B. To assert that A is not B only denies one of the possible conditions. The proposition, "If the trigger is pulled the gun will go off," mentions only one out of several contingencies under which the gun may go off; and if the trigger is not pulled, the gun may yet go off from a sudden shock, or because heat is applied to the barrel. And for the same reason, if the gun does go off, we cannot affirm with absolute certainty that the trigger was pulled; since one of the other causes may have operated.

If we express the hypothetical premise in a categorical form (see Chap. IV. § 1), we see that to deny the antecedent involves an illicit major. Thus:

All cases of A being B are cases of C being D,
This is not a case of A being B,
∴ This is not a case of C being D.

While to affirm the consequent is equivalent to committing the fallacy of undistributed middle. Thus:

All cases of A being B are cases of C being D,
This is a case of C being D,
∴ This is a case of A being B.

Some logicians, *e.g.*, Kant and Hamilton, have denied that the hypothetical syllogism is a case of mediate reasoning at all. They say that it is a case of immediate inference. But the assertion that "If A is B, C is D," by no means implies that "A is B;" it only expresses a relation between the two propositions, "A is B" and "C is D." And the assertion that in point of fact "A is B" is a distinct judgment, resting on evidence of its own, quite independent of that on which the hypothetical premise rests.

In addition to the Hypothetical Syllogism just described, in which only one premise is hypothetical, some writers recognize another form in which all these propositions are hypothetical. Thus:

If C is D, E is F,
If A is B, C is D,
∴ If A is B, E is F.

They confine the term hypothetical syllogism to this form; and call the other the hypothetico-categorical syllogism. The new form is, however, only a curiosity, and has little or no practical value. Such syllogisms can be, in nearly all cases, adequately ex-

pressed as categorical syllogisms by reducing the hypothetical propositions to categorical ones.

All cases of C D are cases of E F,
All cases of A B are cases of C D,
∴ All cases of A B are cases of E F.

§ 2. Disjunctive Syllogisms.

A disjunctive syllogism, as the term is used by most logicians, indicates a form of mediate reasoning in which there are two premises, the first of which is a disjunctive proposition, and the second a categorical; while the conclusion is also categorical.

In a disjunctive proposition a certain number of alternatives are presented, say two; if one of these is not applicable the other is. (See Chap. IV. § 2.)

A is either B or C,
A is not B,
∴ A is C.

This is called the *Modus tollendo ponens,* because it asserts an affirmative conclusion by means of denying one of the alternatives.

Some logicians recognize a *modus ponendo tollens.*

A is either B or C,
A is B,
∴ A is not C.

But this is only valid where the alternatives are known to be exclusive. This we must never assume; if we know the alternative to be exclusive this knowledge can only come from special acquaintance with the subject matter, it is not implied by the form of the proposition. Apart from special information we can never infer the negation of one alternative from the affirmation of the other. From the proposition "He is either a peer or a cabinet minister" we cannot infer that if he is one he is not the other; and the vast majority of the disjunctives ordinarily used are of this kind.

Where, however, both of the alternatives are negatives, the *modus ponendo tollens* may apparently occur. Thus

A is either not-B or not-C,
A is B,
∴ A is not-C.

This, however, is a piece of academic subtlety, as, the alternatives being negative terms, the second premise really *denies* one of the alternatives, and the conclusion asserts the other. It is simply the *modus tollendo ponens* with negative (infinite) terms for the alternatives; and would never be used in actual argument.

The only rule for the disjunctive syllogism, beyond what is contained in the definition of it, is this:—To affirm one of the alternatives, deny the rest.

§ 3. Dilemmas.

A Dilemma is a syllogism with a major premise consisting of two hypothetical propositions, and with a disjunctive minor premise. Or, more exactly, "A dilemma is a syllogism with a compound hypothetical major premise and a disjunctive minor."

Many forms can be found in works on Logic. Some are here given:

(1) *Simple Constructive dilemma.*

If A is B, C is D ; and if E is F, C is D ;
but either A is B, or E is F,
∴ C is D.

(2) *Complex Constructive dilemma.*

If A is B, C is D ; and if E is F, G is H ;
but either A is B, or E is F,
∴ Either C is D, or G is H.

(3) *Simple Destructive dilemma.*

If A is B, C is D ; and if A is B, E is F ;
but either C is not D, or E is not F,
∴ A is not B.

(4) *Complex Destructive dilemma.*

If A is B, C is D ; and if E is F, G is H ;
but either C is not D, or G is not H,
∴ A is not B, or E is not F.

The third form is rejected by Whately, Mansel, and others, who define the Dilemma as having not "a

compound hypothetical major premise," but "a hypothetical major premise with *two antecedents;*" because the major premise in a Simple Destructive Dilemma, though compound, has only one antecedent.

These examples will show what is meant by the *horns of a dilemma.* Two alternatives are presented in the minor premise, on one or other of which the opponent is impaled. Such arguments are sometimes very effective from a rhetorical point of view, but in Logic have very small value. They may put an opponent to momentary silence, but can seldom convince him. The alternatives of the minor premise are seldom absolutely exclusive; some third thing is usually possible which will enable us to escape both "horns."

Thus, in the following simple constructive dilemma,

> If I meet Jones, or if I do not meet him, I shall lose an afternoon,
> I must either meet him or not,
> Therefore, I shall lose an afternoon,

the possibilities of the case are not so simple as they look. It is true that I must either meet Jones or not, but the alternatives do not exhaust the possibilities of nature. A note or a telegram, or even a meeting with a third person may prevent the loss of time.

If the major premise be absolutely true, and the minor absolutely exhaustive also, the dilemma is formally valid; but these conditions usually exist only when the most extravagant suppositions are made, and the examples usually given are grotesque absurdities

of no value whatever. They suppose a simplicity of conditions which does not actually obtain.

Such a dilemma is that given by the historian Aulus Gellius, and frequently quoted in the Logics. Euathlus was a pupil in rhetoric of Protagoras the Sophist; and they arranged that the second half of the fee was to be paid when Euathlus won his first cause. Euathlus appeared in no hurry to begin his professional career, and Protagoras sued him, and addressing him before the judges said, "O foolish young man, don't you see that in any case I must gain my point; for if the court decides for me, you must pay; and if the court does not, you have won your first cause, and must pay according to the terms of the agreement." Euathlus answered, "O most wise master, if the judges give sentence for me, I am acquitted; and if they give sentence against me, I am free by the terms of our bargain, for I have lost my cause." The judges, it is said, regarded the arguments on both sides as unanswerable, and postponed the case *sine die.*

As a matter of fact the difficulty is only a difficulty here and now, and assumes that everything in the world will remain *in statu quo.* There is no real deadlock except for the moment; *solvitur ambulando.* Sooner or later Euathlus will probably plead, and if he is sufficiently well trained to win a case, he must pay; but if he never pleads, or never wins, he gets off by the terms of the agreement. Mr. Welton makes another suggestion, "The most reasonable [solution] seems to be this: As Euathlus had until then won no

case, the condition of the bargain was not fulfilled, and the judges should have decided in his favour. It was then open to Protagoras to bring a fresh suit, when the judgment must have gone against Euathlus."

The plan of meeting a dilemma by another dilemma is a purely rhetorical device, and has no logical efficacy. When the Athenian mother advised her son not to enter public life, because if he acted justly men would hate him, and if he acted unjustly the gods would hate him, she was met by the rebutting dilemma, "But if I act unjustly men will love me, and if I act justly the gods will love me." This is clear enough, but a moment's reflection shows that the love of some men and the hate of others may be compatible with the love of the gods, and generally, that any course will involve some advantages and some disadvantages. The woman overlooked some of each, and the son overlooked the others.

CHAPTER XII.

QUANTIFICATION OF THE PREDICATE AND EQUATIONAL LOGIC.

§ 1. Quantification of the Predicate.

In the proposition, as commonly stated, the quantity of the predicate is not explicitly stated. While we know whether all of the things denoted by the subject are referred to, or whether only some-at-any-rate, there is no mark of quantity affixed to the other term. This doubtless arises from the fact that while the subject is ordinarily understood in denotation, the predicate is ordinarily understood in connotation. (See Chap. III. § 7). But for purposes of inference it is often necessary to understand both in denotation; for instance, in the process called conversion.

It now becomes important to know the quantity of the predicate. We have to bear in mind that the predicates of affirmative propositions are undistributed, while those of negative propositions are distributed. This, however, introduces a possibility of error, which (however slight in theory) is in practice responsible for many mistakes in reasoning. Consequently Sir

W. Hamilton—or perhaps Dr. Baynes—proposed that the quantity of the predicate should be always explicitly stated.

The effect of this reform would be, Hamilton urged, to greatly simplify Logic, by making the proposition an equation, by reducing all conversion to simple conversion, by doing away with the complications of the syllogistic figures (for the mere position of a term as subject or predicate will make no difference to its quantity), and the reduction of all syllogistic axioms to one statement, that, "In as far as two terms either both agree, or one agreeing the other does not, with a common third term; in so far these terms do or do not agree with each other."

§ 2. The New Propositions.

In place of the four Aristotelian propositions, A, I, E, O, quantification of the predicate gives us eight, usually indicated by the letters here placed in brackets after them:

All S is all P	(U)
All S is some P	(A)
Some S is all P	(Y)
Some S is some P	(I)
No S is any P	(E)
No S is some P	(η)
Some S is not any P	(O)
Some S is not some P	(ω)

There is an uncertainty as to whether "some" is

here used in the Aristotelian sense (= "some at least, it may be all"), or as negativing "all" as well as "none." If the former meaning be taken, so much vagueness is left that the proposition in the quantified predicate is just as little an equation as the unquantified proposition.

Some S is some P

may mean any one of the following statements:

Some (but not all) S is some (but not all) P,
All S is all P,
Some (but not all) S is all P,
All S is some (but not all) P.

If the latter or more precise meaning is taken, and "some" is understood to imply "not all," and this is the view accepted by Baynes and by Thomson, Hamilton's most important logical disciples, we get, on the whole, a more satisfactory interpretation; but yet one with many difficulties.[1]

In dealing with any particular case, it will be necessary to ask whether the "some" is to be taken in the Aristotelian sense or the limited sense. And we shall thus have two sets of eight forms to deal with. Students will notice that nearly all attempts to simplify the scholastic Logic end in producing a far greater complication.

[1] Thus Mr. W. E. Johnson, of King's College, Cambridge, has shown that on this supposition an ω proposition is really equivalent to a U; that "Some S is not some P" means that "All S is all P." For particulars see Keynes, "Formal Logic," p. 299 (2nd edit.), and Welton, "Logic," i. 225.

In point of fact, one of the new propositional forms (ω) has obviously little or no meaning; it asserts nothing of value because it denies nothing. A proposition which is compatible with several other forms differing in quantity and quality is of practically no value. "Some S is not some P" is compatible with "All S is P," and even with "All S is all P." "Some water is not some H_2O," means that certain portions of water (say, what is in this jug) are not certain portions of H_2O (say, what is in that basin). Yet all water is H_2O, and all H_2O is water. We can assert ω even of a tautologous judgment; "Dogs are dogs," but "Some dogs are not some (óther) dogs."

The proposition symbolized by η has little more value. To exclude S from some part of P is compatible with the inclusion of S in other parts of P; "No S is some P" is therefore compatible with "All S is P," if some excludes all. "No dogs are *some* quadrupeds" (*e.g.*, tigers), is true, even although "All dogs are quadrupeds."

The truth is, that as negation means exclusion, all negation involves a tacit reference to a *whole* from which the subject is excluded. And even if we employ "some" with the predicate, we no longer use it to mean "some unspecified part" of P (or "some unspecified part, perhaps all," if we adhere to the Aristotelian meaning); but we use it to mean a "certain specific part" of P. It is no longer a mark of vague quantity, but a demonstrative adjective meaning "this part," or "that part" of P.

In "Some S is all P" (Y), the "all" prefixed to the predicate seems to have lost the distributive force of "all" which is usual in Logic, and to have taken a collective or conjunctive meaning. "All cats are quadrupeds," means any cat, or every cat, is a quadruped; the *all* is used in a distributive sense. "Some quadrupeds are all cats" does not mean that "Some quadrupeds are *any* cats," but that they form the whole class of cats. At the same time, such a form is sometimes used. The exclusive proposition, "Only S is P," means that some S is included in P, and that there are no P's which are not S's; that is "Some S is all P." But as has already been shown (Chap. III. § 4), this is really an exponible or complex proposition meaning "Some S is P," and "All P is S." The U proposition is open to the same objection. It really means that "All S is P," and "All P is S."

Neither U nor Y (nor in fact any of the Hamiltonian forms) can be used in the sense in which we commonly do use our propositions; viz., understanding the subject in denotation and the predicate in connotation. While we may grant them (unlike η and ω) some place in an exposition of propositional forms, we see that "the addition of U and Y does not tend towards simplification, but the reverse; and that their full force can be expressed in other ways" (Keynes).

§ 3. Symbolic Logic.

We have seen that for purposes of inference we may fix our attention entirely on the denotation of terms. A proposition may be looked on as a statement that a given class S is included in or is excluded from another class P. Though this is not the primary and natural meaning of the proposition, it is often convenient to adhere to it. Thus the rules of immediate inference and of the syllogism are usually justified by reference to the denotation exclusively.

This point of view being developed, it tends to (1) an equational treatment of propositions, such as the quantification of the predicate aims at; and (2) a quasi-mathematical treatment based on the doctrine of combination.

If there be taken two terms, A and B, we can obviously form four classes: A B, A not-B, B not-A, not-A not-B, or (writing $\bar{A}$ for not-A) A B, A $\bar{B}$, $\bar{A}$ B, $\bar{A}$ $\bar{B}$, or (writing a for not A) A B, A b, a B, a b. We can express any proposition by declaring that one or more of these classes is occupied or is empty. Practically it is found most convenient to express a universal proposition by denying that certain classes are empty, and a particular by affirming that certain classes have some occupants. Thus: "All A is B" is written $A\,\bar{B} = 0$ (or $A\ b = 0$), which denies that the compartment assigned to the class made up of A's which are not B's, contains anything; and "No A is B" is written $A\ B = 0$.

The symbol now most frequently used to indicate that a compartment has some occupants is the letter v, so that "Some A is B" is written $A\,\bar{B} = v$; and "Some A is not B" is written $A\,\overline{B} = v$ (or $A\,b = v$).

§ 4. The Logical Alphabet.

Where there are three terms (with their *negatives*) there will be eight possible combinations, none containing a term and its negative, but each containing each of the terms *or* its negative.[1] For each of the four groups previously mentioned A B, A b, a B and a b, may be qualified by either C or c. Similarly, if there are four terms (with their negatives), there will be sixteen combinations. What these are will be most easily seen by writing A eight times, and $\overline{A}$ (or, as a more convenient negative, a) eight times; then write alongside of the first four A's four B's, followed by four b's side by side with the next four A's, then four B's again and four b's again. Then write C twice, and c twice alternately; then D and d alternately.

A B C
A B C
A B c
A B c
A b

[1] This is not an ordinary case of combinations. If the conditions mentioned were not made, there would be twenty possible combinations according to the well-known formula.

A b
A b
A b, etc.

When completed, the sixteen combinations will run as follows:

A B C D	a B C D
A B C d	a B C d
A B c D	a B c D
A B c d	a B c d
A b C D	a b C D
A b C d	a b C d
A b c D	a b c D
A b c d	a b c d

They form what is called by Jevons the *Logical Alphabet.*

Each of the general propositions negatives one or more of them; and these negatived combinations may be obliterated. The combinations which remain are to be collected and some general expressions found for them. The method is throughout one of elimination or exclusion. Any combination at variance with any statement in the premises must be omitted.

To take an easy example worked out by Jevons ("Principles of Science," p. 100).

"All metals except gold and silver are opaque; therefore, what is not opaque is either gold or silver or is not metal."

Let A stand for metal; B for gold; C for silver; and D for opaque.

The premises may be expressed thus:

$$A\ b\ c = A\ b\ c\ D \qquad (1)$$
$$B = B\ d \qquad (2)$$
$$C = C\ d \qquad (3)$$
$$B = B\ c \qquad (4)$$

And as gold and silver are both metals:

$$B = A\ B \qquad (5)$$
$$C = A\ C \qquad (6)$$

Now, first, it is obvious that we may do what in algebra would be called multiplying both sides of an equation by the same thing. Thus, if all M's are N's, all L M's are L N's; and if $A = b$, all $A\ C = A\ b\ C$.[1] Jevons calls this inference by *added determinants*.

Secondly, it is obvious that any term which contains a contradiction must disappear. Thus $A\ a = 0$, and $A\ B\ C\ c = 0$.

And, thirdly, it is obvious that to repeat the same letter makes no difference to a term. A "red, red house" is a "red house;" an "immortal, immortal, immortal poem" is "an immortal poem." In other words, $A\ A = A$.

[1] This requires some qualification. It is only true on condition that the new qualifying term introduced on both sides has exactly the same meaning on both sides, and comparative words (such as "little" and "large") are liable to change of meaning according to the substantive with which they are placed. They have not always the same denotation. To use the old example, a flea is an animal; but we cannot infer that a large flea is a large animal.

Determining both sides of equation (1) by d (or, as we may call it, multiplying both sides by d), we get :

$$\text{A b c d} = 0 \qquad (7)$$

In the same way (using D for factor) from (2) and (3) we get

$$\text{B D} = 0 \qquad (8)$$

$$\text{C D} = 0 \qquad (9)$$

From (4), (5), (6) we get

$$\text{B C} = 0 \qquad (10)$$

$$\text{a B} = 0 \qquad (11)$$

$$\text{a C} = 0 \qquad (12)$$

We must now strike out every combination which contains any of the expressions we have found = 0. All these compartments of the universe are declared empty. Thus the only combinations left are A B c d, A b C d, A b c D, a b c D, a b c d. We only want to know about d (not opaque things). We find that the only combinations which are d, are A B c d, A b C d, a b c d ; that is, gold metal, silver metal, and things which are not gold, and not silver, and not metal. Q. E. D.

§ 5. Other methods of working problems.

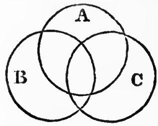
When there are five terms we shall have thirty-two combinations, and when there are six terms, sixty-four. To write out all these afresh each time we have to work a problem, involves a great deal of labour.

Jevons suggests the use of a *logical slate*, that is a slate on which the combinations of letters up to five or six terms are engraved. This will serve for all problems alike. "The conditions of the problem can then be written down on the unoccupied part of the slate, and the proper series of combinations being chosen, the contradictory combinations can be struck out with the pencil." He found such a slate of considerable utility (" Principles of Science," p. 96).

Another plan is to have the combinations printed ; two or three hundred copies may be obtained for a shilling or two.

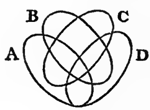
A method which I have found serviceable may be described here. Confining our attention, for simplicity's sake, to four terms, let us copy down in one column the sixteen combinations given above, on p. 142, on a ruled paper. Take a slip of stiff paper and cut out a rectangular space, which will just show the first eight combinations, all which contain A. If this is applied, all the combinations containing *a* disappear. Now take another slip, and cut out two rectangular spaces, half the size of the former, in it, so as to show only those combinations containing B. Make similar slips for C and D. The last will show every other combination only. On the four slips write A, B, C, and D, respectively. By superposing any one, or more than one, of these, we can show or we can obliterate all the combinations containing a given term or terms. This will very much lessen the labour of deleting those combinations which are destroyed by the premises.

Diagrams may often be used with advantage. For three terms we want a diagram like this:



This gives us every possible combination of A, B, C, and their contradictions. The parts which are found non-existent, according to the information given in the premises, may be shaded out. What is left will represent the possible combinations.

The diagram for four terms is as follows:



For more than four terms the diagrams are too complicated to be of any practical service.[1]

Besides the Logical Alphabet and the diagrammatic method, we may employ various directly symbolic methods. These are too difficult to be even indicated in such a work as this ; and the student is referred to Dr. Venn's "Symbolic Logic." All, however, proceed on the same principle as the logical alphabet. All possible classes which can be constituted by a given set of terms have to be taken into account, and the

[1] See Dr. Venn's "Symbolic Logic," chap. v.

classes which are impossible, according to the conditions laid down by the premises, are rejected.

The student should exercise himself in trying to solve easy problems by means of the methods explained in the last two or three sections. Some questions will be found in the Appendix to this volume.

CHAPTER XIII.

INDUCTION.

§ 1. Induction.

INDUCTION is the process of inference by which we get at general truths from particular facts or cases. After examining the cases A, B, C, D, etc., in which the phenomenon x is accompanied by the phenomenon y, we infer that all instances in which x occurs will also exhibit y. It proceeds from singulars or particulars to universal propositions, and it is almost entirely material in character; that is, it depends on the relations between things, rather than on the general form in which we think or express our thoughts.

Like the word "deduction," the word "induction" is used to express both the process of inference and the result of the process. When we speak of *an* induction, we generally mean the latter, that is, the general statement arrived at by inference.

Universal propositions may of course be got at by deduction from some previously known propositions of a universal character. But the series must have a beginning; and we must come at last either to intuition or to induction to furnish us with ultimate premises.

By *intuition* is meant a process of immediate knowledge, where the mind, on being confronted with a statement, or with facts, recognizes without conscious inference the truth underlying. The axioms of mathematics are probably first apprehended in this way. Whether such a power exists at all in the mind has been doubted, and at any rate the greatest part of our ultimate truths clearly come from experience of facts, and are due to a process of unconscious induction.

Almost every science consists partly of universal propositions arrived at by induction, and partly of deductions from these. In some sciences the inductive part is comparatively small, and the deductive part very large ; such sciences are Physics and Political Economy. In other cases, the deductive part is reduced to a very small amount, and the science mainly consists of general truths, which have been discovered by means of induction, as with Botany and Zoology.

A great deal of inference from known cases to new cases takes the form which Mill calls "reasoning from particulars to particulars," and for which Jevons suggested the name *traduction.* A child finds that several little frogs which he touches are cold, and he thereafter expects that other frogs will prove cold if he gets them into his hands. He does not consciously formulate any conclusion of this kind ; there is simply the unconscious expectation of which he is not aware, but of which he would become aware if he met with a contradictory instance. If he begins to think about

the matter at all, he will probably be driven to put it into the form of a universal proposition, "All frogs are cold."

The process of generalization, which is the essential feature of induction, can only take place safely under certain conditions, and it is the business of Logic to indicate those conditions. As, however, the process does not depend on the general forms of our thinking, but on the varying circumstances of things, Logic cannot prescribe the conditions of valid inductive inference as minutely and exactly as it does the conditions of valid deductive inference. Inductive Logic tends to become more an *applied* science than does deductive Logic.

§ 2. Perfect or Formal Induction.

There is, however, an exception to the statement that Induction is not a formal process, but depends mainly on the nature of things. If we enumerate *every* case in which a phenomenon x occurs, and find that in *every* case x is accompanied by the phenomenon y, we may conclude with formal certainty that "All cases of x are y." Every single Cabinet Minister being found to be a Home Ruler, we may safely infer (whether we quite know what "Cabinet Minister" or "Home Ruler" means, or not), that "All the Cabinet Ministers are Home Rulers." The new statement adds nothing to our knowledge of facts, but it puts our knowledge in a slightly different form, by con-

necting the predicate with one general term, "Cabinet Minister," instead of with a number of separate singular terms. It is a convenience to be able to sum up a group of fifteen or sixteen separate propositions in one. There is inference, but the inference is of the slightest character, next door to a "mere verbal transformation." Such inferences are called *perfect* inductions.

They may be represented syllogistically, but not in a mood which is recognized in Aristotelian logic. Thus:

Cases A, B, C, D are Y,
Cases A, B, C, D are *all* the X's,
∴ All the X's are Y.

Here the mood is A A A in the third figure, in which it is usually invalid. But in this case the middle term is distributed in both premises—being a collection of singular terms—and the minor term is properly distributed in its premise, which is a U proposition.

The great bulk of inductions belong to the class of material or real inductions, in which the subject of the universal proposition refers to fresh cases never actually examined. These are of much greater value than the former, but are never of the same distinct certainty. Hence they have been called *imperfect* inductions.

It may be observed that imperfect induction cannot be represented syllogistically. Various attempts have been made to force it into a syllogistic shape, but all

are unsatisfactory. If we take a syllogism of the form just given we get a false minor. Cases A, B, C, D do not now form *all* the X's, but only a very small portion of them. If, on the other hand, we put the reasoning in the following form, we transfer to the major premise the whole of the inference. That is, the induction has really taken place before we profess to begin it.

> All A's which I have observed, and all which I have not observed, are B,
> The two together are all A's ;
> ∴ All A's are B.

The formal process simply adds together the two classes. The real inference is that by which we reach the major premise.

Mill, however, held with Whately, that all inductive arguments may be thrown into the form of a syllogism. The suppressed major in the case of ordinary inductions is, "What is true of the instances A, B, C, etc., is true of all the instances." This in turn leads, by a series of syllogisms, to a syllogism which has for its major premise the principle or axiom of the Uniformity of Nature. This is the "ultimate major premise of all induction." It is curious that Mill, who held that the syllogistic process was not really an inference at all (Chap. X., § 2), should also maintain that the inductive process is essentially syllogistic. Both deduction and induction must therefore be only a process of verbal transformation, and there can be no such

thing as real inference, except from particulars to particulars.

§ 3. Laws and Uniformities.

The result of generalization is called a Law. "Whenever over a given range one fact is accompanied or followed at any interval by another fact, we call this a *uniformity*," says Mill. A *Law* is the expression of a uniformity by means of words or symbols.

The word Law is used in two or three distinct senses. When we are talking of politics or jurisprudence, a law means a general command given by a government to its subjects. It is distinguished from particular orders, or acts of the executive, by being addressed to every person belonging to a certain category. In this sense a general rule of a school is a "law," or a rule binding all the boys of a particular form; while an order addressed to Brown or Jones is not a law. This idea of general command was transferred to theology, and from theology to science. The creative fiat was regarded as a law, which of course in any exact sense it could not be. Only in a rough analogical sense could the tendencies found in the natural world be regarded as due to laws promulgated by God. A law implies intelligence, and the possibility of obedience or disobedience. It was never open to a stone to obey or disobey the "law of gravitation." Nature may be the expression of God's will;

but law and obedience are alike impossible without rationality.

Modern science has got rid of the theological implication in the idea of "laws of Nature." It suggests no opinion as to their origin. Whether theist, or atheist, or agnostic in his theological opinions, the man of science, as such, means nothing by Law except the expression of a uniformity which has been detected among natural phenomena. The element of command in the original idea has gone; only the element of generality remains.

Strictly speaking there are no laws of Nature or in Nature. What we call a law of Nature is our expression of what we believe we have observed. It is a mental product, due to abstraction and generalization. Man does not discover laws already immanent in Nature; he invents abstract expressions to describe what he observes. These laws are always, in one sense of the term, hypothetical. They imply that if the objects of a class conform to a recognized type they will have certain qualities. Nature makes individuals differing in all kinds of ways. Man "sorts" these individuals, and although he ascribe objective validity to his classes, they are, as Locke long ago showed, the creation not of Nature, but of man.[1] And thus the so-called laws of Nature, which depend on these classifications, are never entirely objective. Uniformity is read into Nature by man. It is just as true to say that Nature is infinitely various as to

[1] See Chap. XX. below.

say that she is entirely uniform. She is at once both and neither. Difference and likeness are both mental realities. They are relations which exist only for conscious minds.

Uniformities among phenomena are of two main kinds, those of Sequence and those of Co-existence; "Every X is followed by Y," or "Every X is accompanied by Y."

The first class embraces two subdivisions—those of mere sequence, and those of causation. It is the latter which are of chief importance. Laws of succession are the most valuable of all truths, says Mill; and of these the laws of causation are the only ones which are perfectly "indefeasible" and "co-extensive with human experience."

Uniformities of co-existence include the truths of number and space—unless indeed these are to be regarded, not as observed uniformities, but as intuitively perceived relations—and the properties of natural substances whether organic or inorganic. These uniformities of co-existence are less general and less certain than those of sequence. "Uniformities which obtain in the co-existence of phenomena," says Dr. Bain, meaning those in which causation is not known to play a part, "are entitled to reception only as empirical laws; and are not to be presumed true except within the limits of time, place, and circumstance in which the observations were made, or except in cases strictly adjacent" ("Logic," ii. 122).

The student should notice that we often put into

the language of co-existence what is really a matter of sequence. "We term the snake 'deadly' just as we term it 'supple' or 'many-ribbed,' thus transferring the occasional sequence [viz., death following on its bite] to a place among permanent co-existences" (Venn, "Empirical Logic," p. 92).

§ 4. Laws of Nature and Empirical Laws.

The term Law of Nature, in its original quasi-theological sense, was generally confined to those great general uniformities which are found everywhere and always. Thus the presence of gravity was regarded as due to a law of Nature; but the presence of thirty-two teeth in an adult human being, or the acidity of lemons, was not so regarded. God was perhaps supposed to have separately willed the great facts and tendencies which men discovered to be universal, or almost universal; while the smaller facts of Nature were thought to have come into being as a consequence of the former, without any further creative act.

Modern writers keep up something of this distinction, without the theological suggestion underlying it. While the expression, "Law of Nature" is sometimes loosely used for any observed uniformity, it is more commonly kept for the most ultimate, simple, and fundamental laws, of which the more complex and concrete laws are special cases. Thus, the movement of a planet in an ellipse is derivable from, and can be

resolved into, the law which expresses the action of gravitation, and Newton's first two laws of motion.

Such a particular or special law, when shown to be dependent on more ultimate laws, is called a *derivative law.*

Bacon called these derivative laws *axiomata media,* that is, truths of moderate generality intermediate between mere facts and the highest generalizations.

The expression of an observed uniformity which has not yet been resolved into simpler and more ultimate laws, but is presumed to be capable of such resolution, is an *empirical law.* It rests only on the evidence of experience (ἐμπειρία).

The most ultimate laws which we know are, of course, not derivative, and they are arrived at from experience; but they are not empirical laws in this specific sense, since they are not presumed to be resolvable into laws still simpler.

CHAPTER XIV.

UNIFORMITY OF NATURE AND CAUSATION.

§ 1. The Ground of Induction.

GENERALIZATION is possible only because Nature is uniform. It is this external condition which makes it possible for us to make material inferences. All inductive laws are particular cases of uniformity, are expressions that the uniformity holds good within a certain sphere. Or, as Hume puts it in his "Inquiry concerning Human Understanding," "All inferences from experience suppose as their foundation that the future will resemble the past, and that similar powers will be conjoined with similar qualities." It covers regularity of sequence (including cases of causation) and regularity of co-existence.

In every real induction (as opposed to "perfect," or formal induction,) there is a leap to the future. We assume that the cases we have not yet examined will resemble those which we have examined. This assumption can never be entirely warranted by experience, since experience can only refer to the cases we have met with. Its justification, as Professor Bain says, is entirely practical, and lies in the fact

that without it we cannot take the slightest step, while "with it we can do anything" ("Deduction," pp. 273, 274).

We must always assume Nature is such that whatever is true in one case of a given kind is true in all similar cases. Whether this truth is ultimately a law of things or a law of mind, or whether the two statements ultimately mean the same, has been discussed by philosophers. Nature is uniform, it would seem, because otherwise experience is impossible. What we mean by Nature is the ordered Cosmos; a mere chaos of unrelated sensations turning up without any order would not be Nature, and would be, in point of fact, inconceivable. Human life and human consciousness could not exist without some degree of uniformity.

The Law of Homogeneity, as we have seen (Chap. VII. § 4), lays down that however different two objects of thought may be, they can always be brought under some higher concept. Likeness always lies hid beneath apparent differences. In any group of phenomena there is likeness, expressible as a law, if we only know where to look for it.

The principle of the Uniformity of Nature includes what is called the Law of Causation, but it includes something more. It implies that the same bundles of qualities go together, but it does not necessarily imply that these bundles go together because they are produced in common. The regularity may not be due to causation at all. There is indeed a growing tendency to assume that all cases of uniformity are due to common origin

and to similar conditions of production. The theory of evolution has done much to justify this belief. The similarities which are observed throughout the organic kingdoms have been traced, with every probability, by Darwin and his disciples, to common ancestry. Even the similarities observed among inorganic things, such as minerals and stars, have been ascribed, with high probability, to the fact of a common origin. It is possible then that all cases of uniformity may be due to uniformity of causation. But we are not at present in a position to prove this, and we may hold that Induction would be possible even if the Law of Causation were unknown.

§ 2. The Law of Causation.

This Law of Causation has been variously stated, but it contains practically three propositions, two of which are necessary to its expression, while the third is necessary if it is to be of any practical value. They are as follows:

(1) That every phenomenon that takes place in time, (or in other words, every event) has a cause.

(2) That the same effects follow the same antecedents.

(3) That the same antecedents do as a matter of fact recur.

The first of these is a part of Leibnitz's Law of Sufficient Reason (Chap. VII., § 5). It is of course not provable. It is an assumption which we are obliged to make, and which we constantly do make in practice.

In so far as we do not make it, or forget it, we suffer in our contact with Nature and Society.

The same thing may be said of the second proposition. This is a part of what we have called the Law of the Uniformity of Nature, but it is an important part of what is implied in the Law of Causation, as generally understood.

There has been an infinite deal of discussion about this extremely important idea of causality, and philosophers are as a rule in marked disagreement with ordinary practical men, and even with men of science, as to what it really means. The logician is not bound to go deeply into this dispute; it will be sufficient for him to find a working definition which will serve practical and scientific purposes, even if this does not profess to be a completely accurate definition.

The notion of cause as held by average unsophisticated people, *e.g.*, by the business man and the ordinary medical practitioner, seems to be well expressed in the words of Reid: "The name of a cause . . . is properly given to that being only which by its active power produces some change in itself or in some other being. Active power, therefore, is a quality in the cause which enables it to produce the effect" (Reid's "Works," Hamilton, p. 603). This view is really a survival of Aristotelian and scholastic teaching, like most of the floating and uncritical philosophy which masquerades as Common Sense.

Hume attacked the common-sense doctrine. He pointed out that no such thing as Force, Power, or

Energy is ever an object of experience. "One event follows another, but we never can observe any tie between them. They seem *conjoined*, but not connected" ("Inquiry concerning Human Understanding," Section VII.). He gets rid of the metaphysical nexus, or bond, of power, and retains mere sequence. Mill developed Hume's doctrine. With him the cause of a phenomenon is the "antecedent, or the concurrence of antecedents, on which it is invariably and unconditionally consequent" ("Logic," Book III., Chap. V., § 6).

As a matter of fact the cause is always a concurrence of antecedents, a *group* of circumstances, all of which must be present to bring about the effect. The cause of the flame seen when a match is struck is not simply the act of striking, but also the composition on the match and on the box, the heat produced by the friction, the presence of oxygen in the air, and many other less obvious conditions besides. We must even include what are called *negative conditions*, such as the absence of wind in the air and the absence of damp on the box. The word cause is applied sometimes to one and sometimes to another of this large group of invariable antecedents, according to the special interest and point of view which we are taking.

If a man is killed by a gun, what is the cause of his death? For the ordinary man it is sufficient to say that the bullet was the cause. But for any special purpose much more is needed. The doctor at the inquest says the man died from a gunshot wound in his neck. A

scientific physiologist would be able to go much further, and point out exactly how the wound brought about the death, whether by shock to the nerves which control the heart and lungs, or by the influx of blood into the air-chambers of the lungs, and so on. The expert in guns would point out how the gun came to explode, and how the missile came to hit the man. The physicist would explain how the explosion was brought about by the expansion of gases contained in the powder. The lawyer would be interested in the exact psychological conditions which determined the pulling of the trigger; the act of the man who held the gun is for him the cause of the victim's death. The moralist would lay most stress on the recklessness or malice which led to the pulling of the trigger; for him the cause of the death lies in the character of the man who held the gun. These events form a chain, or rather a continuous series, all the parts of which are marked off and distinguished by different classes of people in somewhat different ways. What the lawyer calls part of the "effect," the doctor calls the "cause." Instead of a chain of distinct links we have a rope on which portions have been marked off. And different portions of this rope, at different distances, are fixed upon as cause of the man's death.

Keeping in mind the absolute continuity of this series, and the large number of concomitant conditions which at every point are presupposed, we see that if we want to be exact we shall have to say that the whole state of the universe at any moment is the cause of its whole state at the next; otherwise we may accidentally

omit something of importance. It is, for instance, just possible that some electric or chemical condition of the atmosphere, of which we at present know nothing, may be necessary to the explosion of the powder. But such a conception of cause as this is of no practical value. We must omit the vast majority of concomitant phenomena and confine ourselves to what we call relevant or material circumstances, even at the risk of overlooking something important.

The logician, then, takes an intermediate view. He substitutes for the single antecedent of the "plain man" a group of antecedents, and he includes negative antecedents. For his purposes he leaves out of sight the conception of energy passing over from cause to effect, while he does not necessarily deny that such a conception is implied by the idea of cause, if it be thoroughly analyzed.

§ 3. Plurality of Causes.

The second proposition included under the Law of Causation was this, that the same antecedents are followed by the same effects. The same antecedents never do recur, if we take a sufficiently large group. Certainly the same state of the whole universe is never repeated; but we need not go so far as that to see that absolute repetition does not occur. While by increasing the size of our group we secure greater accuracy, we secure it at the expense of practical utility, for the larger the group of antecedents the less frequently will it recur.

Can we turn the second proposition round and say that the same effects are always preceded by the same antecedents?

At first sight it would seem that we cannot. A gunshot wound in the heart always kills a man; but a man may die of ten thousand other things than a gunshot wound in the heart. This is what Mill calls the *Plurality of Causes*, and what Professor Fowler calls the "Vicariousness of causes." It seems obvious enough that the same effect may be produced now by one cause, now by another. But reflection shows that this is only so long as we keep to the loose, popular treatment of cause and effect, which defines the cause with more accuracy than it does the effect. We take the effect, death, in a generic and abstract sense, not discriminating between the different kinds of death; while we take the cause in a specific and determined sense. If we say precisely what sort of death the man died, how he fell, the appearances presented by the internal and external organs, then we shall find that only one cause is possible. "Had we been equally exhaustive in our enunciation of the constituent elements in the aggregate effect as we were in those of the cause, no such plurality would have been possible," says Dr. Venn.[1] And he adds: "So clearly is this recognized whenever it becomes important to take

[1] Dr. Venn's treatment of the logical aspect of causation is by far the clearest and best that I know. Much of this chapter is due to his lectures, his "Logic of Chance," and his "Empirical Logic."

it into consideration, that the whole procedure in a trial for murder, or in any coroner's court, rests upon the assumption that if we are particular enough in our assignment of the effect there is no possibility left for any plurality of causes" ("Empirical Logic," p. 62).

§ 4. Conjunction of Causes and Intermixture of Effects.

In very simple and abstract cases of causation, where certain factors are out of all proportion more important than others in the production of the effect, we may often think of an effect as due to a single prominent antecedent. In considering the movement of a billiard ball on the table, we think only of the progress in a straight line in the direction of propulsion. The total effect is really much more complex; since, if we want to be precise, we ought to take into account the movements of the earth on its axis and round the sun, and even the movement of the whole solar system towards a certain point in the constellation Hercules. Disregarding these, we must not forget that even the movement on which we fix our exclusive attention is really due not merely to the stroke of the cue, but also to the nature of the ball, the cloth, the table, and the floor; to mention no other co-factors.

In abstract inquiries we may confine our attention to only one cause; and thus in mechanics we may speak of a body as moving under the impulse of only

one force. This must not, however, be regarded as a normal type. We cannot in any actual and concrete cases assign a single cause for a single event. All instances of causation are cases of what Mill calls *the interference of causes* and *intermixture of effects.*

Mill probably leaves most of his readers under the impression that normally each cause produces its own effect, and that the intermixture of effects is a peculiar and somewhat unusual occurrence. Whether he actually held this view is difficult to settle; but at any rate it is erroneous.

Two kinds of concurrence of causes are distinguished by Mill. "In the one, which is exemplified by the joint operation of different forces in mechanics, the separate effects of all the causes continue to be produced, but are compounded with one another, and disappear in one total." Such a total effect is called a *compound effect.* "In the other, illustrated by the case of chemical action, the separate effects cease entirely, and are succeeded by phenomena altogether different, and governed by different laws." Such an effect, differing in kind from the results of the separate causes acting independently, is called a *heteropathic effect.*[1]

Mill has overstated his case a little. Even in chemical combinations the separate effects do not "cease entirely." Hydrogen and oxygen exploded together produce water; and although this is very different in some respects from its component gases, it resembles

[1] Mill, "Logic," III. x. § 4.

them in others. The weight, for instance, is the sum of the weights of the components.

But the whole discussion is somewhat unreal; because, if we take at all a scientific view of causation, we can never hope to assign particular effects (or rather particular portions of the total effect) to particular factors. In actual Nature, as opposed to the abstractions of the class-room, a vast number of coöperant conditions are always present, and we can only say with diffidence how far any of them can be omitted. We can never hope to analyze a case of causation into a group of phenomena, A, B, C, D, followed by another group, a b c d, so that A produces a, B produces b, and so on.

CHAPTER XV.

OBSERVATION AND EXPERIMENT.

§ 1. Observation.

ALL our knowledge—with some small and disputable exceptions—comes from experience, either directly by perception, or indirectly by way of inference from perception. Nearly all deductions have for premises propositions which assert matters of fact. Thus the first step in knowledge is due to perception either of facts of external nature, or of facts of consciousness.

Observation means systematic and careful perception. It involves voluntary attention, directed on the object. When we thus purposely attend to an object, the object is ordinarily perceived with greater quickness, vividness, definiteness, and completeness than when we merely just notice it. Features which would be overlooked become clear, and are distinguished from other features with which they are at first confounded. A certain abstraction of attention from other objects is involved in this concentration of consciousness on the given object.

In order to secure the fullest operation of our perceptive powers, some analysis of a complex object

is necessary. Some small object, some special group of phenomena, or some single aspect of an object has to be isolated. This is done to some extent by the act of attention itself, as we have seen; but that it may be carried out more thoroughly, artificial changes are made in the conditions of observation; a flower is removed from the plant, the flower is broken up into its constituent parts, and sections of these parts are made and considered separately. Special instruments are brought to bear in order to effect this analysis and isolation, *e.g.*, the dissecting-knife, the live-box, and the test-tube.

Other instruments are employed for rendering minute phenomena perceptible. Such are the microscope and the telescope; the spectroscope, as used by chemists for purposes of chemical analysis; and the photographic camera, as used by the astronomer to detect stars whose light is too feeble to affect the retina. When quantitative results are required, special instruments are necessary for measuring the extent or intensity of phenomena. The measuring rod, the balance, the chronometer, the barometer, and thermometer are well-known instances of such artificial aids to observation.

Other instruments are used to economize time and attention, by effecting automatic registration of results. The occurrence of direct sunlight is recorded by a burning-glass arrangement, and the photographic camera is employed very largely for purposes of registration.

§ 2. Experiment.

Professor Fowler says that to experiment is "not only to observe, but also to place the phenomena under peculiarly favourable circumstances as a preliminary to observation" ("Induction," p. 38). If we take this definition, we shall not be able to refuse the term experiment, even to the proceedings of the field observer, who plucks a leaf in order to examine it with more exactness than he could do on the bush where it grows. The amateur who takes home a butterfly to identify it becomes an experimentalist, as well as the physiologist who employs vivisection. In fact, we may say that in nearly all real observation—that is, careful, prolonged, and attentive perception—the conditions are to some extent altered to suit the special needs of the observer. Meteorological observatories are placed on lofty mountains and on out-of-the-way islands to secure favourable opportunities not presented elsewhere. When a total eclipse of the sun, or a transit of Venus, is expected, astronomical expeditions are sent to many different parts of the world in order to take advantage of the variety of conditions under which the phenomenon may be seen.

Mill regards the essential difference between observation and experiment as lying in the fact that in the former "we find an instance in nature suited to our purposes," and that in the latter "by an artificial arrangement of circumstances we make one" ("Logic," Bk. iii. chap. vi. § 2). Of course in the strict sense of

the word *we* never make an instance. Nature always does the work, we can only bring about the conditions under which the event takes place. When we say that we produce a phenomenon artificially, we can never mean more than this.

The difference then between observation and experiment becomes one of degree. Where the conditions under which the facts occur are considerably altered, by bringing fresh forces to bear on the thing observed, so that for all observers the phenomena themselves are considerably altered, we call it experiment. Where the conditions are not markedly changed, or where they produce change in the phenomena only for the individual observer, we usually call it observation. If an astronomer goes to the Cape to observe an eclipse, the changes in the phenomena are not considerable, and exist only for him. To an astronomer who remains in Europe there is no change. If a physiologist produces a series of changes in the body of an animal by some operation on the brain, the changes are considerable, and they exist for any competent observer who sees the animal.

All artificial interference with living things produces changes which must be taken into account; so that the phenomena must no longer be regarded as strictly normal. The phenomena observed during dissection and experiment are not the same phenomena as those of normal life; though by the exercise of scientific imagination we may construct the normal phenomena from the hints given us.

The vital relations which give a meaning to the phenomena are only too apt to disappear during the process of analysis.

The chief advantages of experiment over observation are these: (1) We can multiply our instances to almost any extent. Instead of waiting for an infrequent concurrence of conditions, we can at once produce that concurrence; and then the effect follows. (2) We can isolate the phenomena to be studied, so that they are presented apart from disturbing circumstances which distract the attention, and in other ways render the given phenomena difficult to observe. (3) We can vary the circumstances, so that we can clearly separate the necessary conditions from the conditions which are not necessary. (4) We can *prove* by experiment that a given group of conditions is actually the cause, or at any rate one of the causes, of a given effect; since by artificially producing the given group of antecedents we shall in that case obtain the effect. We suspect that the multiplication of a certain microbe in the blood causes a specific disease; we can *prove* that this is so, at any rate sometimes and under some conditions, if when we artificially produce this multiplication of the microbes in the body, the disease is actually produced.

Where experiment is impossible Induction is at a great disadvantage. As we shall see, experiment is the chief means by which generalizations are verified, and Verification is the chief characteristic which marks off scientific induction from futile guessing.

On the special difficulties in the application of this method of verification, see Chap. XVIII. § 3 below.

§ 3. Non-observation and Mal-observation.

In both observation proper and experiment two important conditions are clearly necessary if the results are to be the basis of correct inferences. We must be careful not to overlook anything that ought to be observed; and we must be careful not to believe we observe phenomena which are not there to be observed. We must guard against Non-observation and Mal-observation.

The plain man seldom realizes how difficult the work of observation really is. To him nothing seems easier than to perceive facts which are before us; except, perhaps, to refrain from perceiving what is not before us. In practice, both conditions are difficult to comply with; and only trained observers can as a rule be trusted to accurately perceive and to accurately record their perceptions.

A few hours in a court of law listening to the evidence of witnesses in (say) a commercial case will shake the faith of the plain man. He must come to the conclusion either that at least half the eminently respectable witnesses are lying, or that human faculties of perception are no more infallible than those of inference. Here is another instance. A few years ago, during the "dull season," a controversy was raised in

one of the daily newspapers as to whether eucalyptol would rid a room of house-flies. Some of the correspondents asserted that it did, and some that it did not; and both parties gave their conclusions as the result of personal observation, which as a rule they detailed. The uniformity of Nature seemed to be temporarily suspended in regard to house-flies.

There are, of course, some phenomena which cannot be easily overlooked, *e.g.*, a rash on a patient's face. But there are others which require trained skill to recognize, and sometimes the use of instruments which can only be properly employed by experts; *e.g.*, the sounds caused by certain diseases of the heart. In the latter case, besides delicate senses, which have been thoroughly educated, we want prolonged attention and considerable previous experience. We want what psychologists call preadjustment of attention. We must know exactly what we expect to find.

Everybody who has used a microscope knows that, in order to find a minute object in the field, it is important to have a mental picture of what we wish to see. The same thing is true when we try to recognize a distant object, *e.g.*, a ship at sea, or the face of a friend in a great crowd.

But this very preadjustment of attention involves a special danger. What we expect to see we want to see, or fear to see; and what we want to see, or fear to see, we are liable to see, whether it exists or not. The expectant attention distorts the perception, and illusion follows. The facts become accommodated to

the image which we have formed beforehand. The medical man who expects to find a certain symptom, sometimes thinks he perceives it although it does not exist. Valetudinarians, it is well known, are constantly a prey to imaginary symptoms; their fears produce the phenomena which they dread; and, for just the same reason, the terrified child sees a ghost in a haunted room. The most practised observers have been known to find traces of organic structure in non-organic things. The so-called *Eozoon Canadense* was taken for a fossil organism by Sir W. Dawson, Dr. Carpenter, and other scientific men, though all, or nearly all, recent geologists regard it as inorganic. Professor Haeckel thought he found the characteristics of organic matter in an inorganic slime, which he named in honour of Professor Huxley.

Multiplication of instances under different circumstances, the use of different lenses with the same instrument, altering the light, shifting the stage of the microscope, the employment of other instruments besides our own, can guard us to some extent against such errors. Not less important will be the comparison of our own observations with those of others. The suggestion sometimes advanced by anti-vivisectionists—that when an experiment has once been made, there is no need to repeat it—betrays the most profound ignorance of the conditions under which scientific study must necessarily be conducted.

Merely negative results go for very little. The fact that up to the present I have not noticed a phenomenon,

or even the fact that no one has yet noticed a phenomenon, does not prove that there is nothing to be perceived.

§ 4. Perception and Inference.

All our actual perceptions involve a certain amount of inference, as we have already seen (Chap. I. § 1, above). But we cannot draw a clear line, and say where the data of sense end, and the stage of subconscious inference begins. The data of sense are an abstraction, an ideal which our analysis never reaches. Pure sensation, as Dr. James Ward says, is a psychological myth. What we call sensation is already a complex mental product.

There is a tendency amongst unskilled observers and persons without philosophical training to regard as perception a great deal that is not only inference, but highly precarious inference, which a little psychological analysis would have served to detect.

Thus the redoubtable Mr. Sherlock Holmes misleads his amiable companion, Dr. Watson, by telling him on a certain occasion that "*Observation* shows me that you have been to Wigmore Street Post-office this morning;" when it is clear, from the explanation which he gives, that this is merely a probable inference, based on the fact that he had noted some red mud on Dr. Watson's boot, and that there was some red mud of a similar kind in front of the post-office, and, so far as he had observed, nowhere else in the neighbour-

hood.[1] All that observation could really tell the detective was, at most, that there was some reddish substance adhering to the boot. Everything else was inference.

What a man sees depends on many circumstances. The delicacy of his organs, the attention he pays, the amount of his previous knowledge, the external conditions of observation, all influence the result. A savage brought into a library literally does not *see* the same things as the librarian. He sees strips of red and blue, he does not see books. He does not distinguish the volumes of a series from each other, or from the shelf on which they are placed.

§ 5. Fact.

The word "fact" is used in two or three senses. It means, etymologically speaking, something done (*factum*). What has actually occurred is a fact. From this meaning it is easy to pass to others; and fact comes to mean any phenomenon or group of phenomena actually observed in time or space—an event or a thing—thought of more or less as individual and distinct. In this sense it is opposed to a general truth or law, which is the formulation of a uniformity observed in many separate instances. Such a law is based on the *facts*, that is, the separate observed instances.

What is observed is real; at least it is real if the

[1] "The Sign of Four," chap. i., pp. 11, 12.

conditions of observation are normal, and the observer is normal. Our nearest approach to objective truth lies in the perception of external facts. Even the most intelligent persons differ in their inferences from concrete facts; but it is assumed that they cannot in their perceptions, their senses being supposed to be equally delicate, their knowledge and attention being supposed the same, and the conditions of observation equally favourable. This is not exactly true, but it is on the whole correct. Hence, when we say anything is "a fact," we mean that it is objectively true, true for everybody.

Is it a law or a fact that the tide is caused by the moon and sun? It is a *law,* in the sense in which law is opposed to fact; since this is not a phenomenon given us in perception, but a uniformity inferred from a large number of observations. It is a *fact,* in the sense in which law is not opposed to fact; since the law or uniformity is true.

CHAPTER XVI.

THE INDUCTIVE METHOD.

§ 1. Method.

By method we mean, as Hamilton says, "the arrangement and elaboration of cognitions according to definite rules, with the object of conferring on these (the cognitions) a logical perfection" ("Logic," § lxxx).

Men of science first get at their facts, from their facts make their generalizations, and from these make their deductions. It is the business of Logic to point out the general conditions to which these processes must conform if the results are to be correct. Our reasoning powers act to a large extent automatically, and apart from direct volitional control. Logic does not so much undertake to show how we must reason, as to show the conditions which will preserve us from serious error. Thus in Deduction, Logic does not prescribe the syllogism as a necessary form if we want to reason properly; but as a convenient form for stating our reasonings if we wish to test them.

The purpose which we have in hand will determine the special kind of arrangement which ought to be adopted. The method of exposition is not necessarily

the same as the method of discovery and proof. The order in which our facts and ideas shall be exhibited is a matter of the greatest importance. The dependence of one thought on another can only be adequately seen when care has been bestowed on making clear the implied relations between the different propositions to which we have been brought to assent. In the confused thinking of the plain man about politics or theology there is frequently a startling inability to differentiate premises from conclusions, hypotheses from axioms, and definitions from enunciations.

Any arrangement of the material of thought, so as to exhibit its logical relations, and thus render the processes of thought easy, systematic, and safe, may be called a method. Consequently the word is used in many different connections. We may speak of the method of Induction, the method of Deduction, the method of Discovery, Mill's methods, Bacon's method, the analytic method, the synthetic method, the experimental method.

By a natural transition we may apply the term to any rule for dealing with our material in such a way as to give us valid and useful inferences. Thus we speak of the method of Least Squares, and so forth.

§ 2. Analysis and Synthesis.

Analysis means the division of a whole; the breaking of it into parts in order that it may be more com-

pletely examined and understood. This may be a physical process, as in chemistry, or a mental process, as in psychology. In the one case we obtain separately the simpler constituents of a compound; in the other we obtain in ideal (not actual) isolation the simpler constituents of a mental state.

Synthesis means the placing together of simpler wholes with the purpose of producing a new whole of a more complex character. When two physical wholes, such as a portion of sulphuric acid and another of copper, are combined to form sulphate of copper or blue vitriol, we have synthesis. The construction of a box out of six pieces of wood is a case of synthesis. So is the formation of a compound notion, such as "black board" or "ecclesiastical monarchy," by putting together the simple ideas of which it is the sum.

A little reflection will show us that synthesis and analysis are usually found together. For the purpose of synthesis a preliminary analysis is usually requisite; and so too analysis is often possible only by means of simultaneous synthesis. In a prolonged effort of construction or of reasoning, both processes may have to be performed many times. Thus the neat and properly shaped boards of which the box is made, and the pure metallic copper, have to be separated from what is found with them. A process of analysis has to be gone through before the synthesis is begun. The idea "ecclesiastical" and the idea "monarchy" are the results of many successive acts of analysis and synthesis.

If we are to completely understand the nature and the conditions which produce a given whole, it is obvious that we need both processes. We must be able to pull it to pieces and to build it up again. By means of synthesis we verify the results of our previous analysis.

And we must remember that the material elements which we can perceive by means of the senses are not the only elements which a complete analysis will yield. The human body is something more than a pile of anatomical scraps. A muscle is something more than the carbon, oxygen, nitrogen, and so forth, into which the chemist's analysis resolves it. The original vital union in which the elements are bound cannot be always imitated or reproduced by the artificial synthesis of the scientific man. Something escapes him. As Wordsworth puts it—

> "We murder to dissect."

The complete natural synthesis is very different from the clumsy and incomplete synthesis which science is able to produce.

All abstraction is primarily a process of analysis. All comparison is primarily a process of synthesis. "We resolve analytically the presentation-complexes [objects of perception] of our concrete experience only in order to establish certain relations among them. The most appropriate term for all such conscious 'relating' or discernment of relation is comparison." "Comparison thus involves abstraction; abstraction,

even in the case of a single object, may be said to involve the rudiment of comparison."[1]

An attempt is sometimes made to identify induction with analysis and deduction with synthesis. This is to be avoided. Synthesis and analysis are, of course, much wider terms than induction and deduction. Definition, Division, Judgment, Immediate Inference, etc., all involve both analysis and synthesis. So do Deduction and Induction.

A deductive syllogism may be regarded as a process of synthesis because it unites the contents (or matter) of two propositions in a third. But it is also a process of analysis, because it splits up the meaning of the middle term. Thus the syllogism,

All birds are two-legged,
All linnets are birds,
∴ All linnets are two-legged,

analyses the meaning of bird, or that group of attributes which we associate with the name bird, and asserts of linnets part of that meaning. Viewed, however, from the point of view of denotation, there is synthesis. The class linnets is now added to the other classes which compose the superior class—two-legged things.

Induction in the same way may be regarded as a process of analysis, because it breaks up the whole group of attributes implied by the name (connotation),

[1] See Sully, "Outlines of Psychology," pp. 244 *seq.*, 251 *seq.*, 290 *seq.*

or ordinarily found in connection with those that are so implied. We assert of all birds part of what we perceive in the case of all those we examine. But in doing so we have synthesis as well, because we group all the individual objects into a whole, "All birds."

Since the formation of every concept is at once a process of analysis *and* of synthesis, and since the formation of a judgment also involves both processes, it is idle to regard any given process of reasoning as essentially analytic, or essentially synthetic.

§ 3. Classification in Induction.

We start with a number of separate propositions, each of which we will suppose proved. "The thing A has the quality Q," "the thing B has the quality Q," "the thing C has the quality Q," etc. Our first need is to group A, B, C, etc., into a class, so that we may know exactly what group of attributes ordinarily go together, and that we may be able to identify this group wherever we see it. We recognize A, B, C, etc., as belonging to a class S already acknowledged, or we construct a new class, which includes them and all other things exactly resembling them.

Induction, then, presupposes a classification. Sometimes the class is ready to hand as the result of previous observation and inference; sometimes a new class has to be formed. Mill did not quite adequately recognize the importance of this step, but it has been emphasized by Whewell (whose knowledge of the

history of science made him Mill's superior in questions of this kind), by Dr. Venn, and other logicians.

Induction, says Dr. Bowen, involves the "correctness of the preceding classifications that have been made of the objects of sense." The more accurate are our Definition and our Division, the more safely can inductions be made. The classifications of the abstract sciences, such as geometry, are so precise, that universal propositions may be safely made from the examination of one instance. What is found true of a single circle is true of all; because the mathematician's circle is not the rough figure he draws, but the ideal circle which absolutely conforms to the definition. Even the chemist has something of the same certainty. As long as he means by "water," H_2O, whatever he finds true of one drop will probably be true of all under precisely similar conditions; only that he cannot be certain whether a given property which he discovers in a special sample of water belongs to perfectly pure water (H_2O) as such, or only to the present sample, which consists of H_2O, *plus* some unknown impurities.

When the class is not a recognized one, the case presents many difficulties. Let us suppose a medical man, at the beginning of the influenza epidemic, confronted with a number of cases in which he notices a very yellow tongue. In many respects the cases differ. He sees some resemblance, however, and on the supposition that he has never heard of influenza, or, at any rate, knows nothing about it, he will have to try to form the conception of a disease, which shall

always present certain symptoms, or, at least, several out of a group of symptoms. Let us suppose that he now consults other medical men, or reads the "Lancet," and learns to call this disease, so marked out, "influenza." He has introduced what Whewell calls an "appropriate general conception," and has fixed it with a name.

§ 4. Generalization.

The next step will be to see whether the symptom, yellow tongue, is always present or not. This will enable us to affirm, or to deny, the statement that "all influenza patients have a yellow tongue."

The possible truth of the judgment is suggested by the few cases examined which first aroused interest in the matter. As a matter of fact probably the medical man would begin with an indefinite class, not waiting to get an exact conception, but saying, "All the patients suffering from this undetermined but striking disease have yellow tongues." The mind, in other words, starts with a hypothetical class, and makes a hypothetical generalization about the members of it.

Then as he sought for evidence of the generalization, he would gradually define his class more accurately. But sometimes at any rate the class is found ready made; and sometimes it is marked off clearly as the first step before generalizing.

In a few cases of influenza he finds that the patients

have yellow tongues: will this always be found to occur? Is Q a quality of *every* S?

The test of this lies in the examination of instances.

Suppose only those instances are examined which come spontaneously to hand, and that no contradictory cases are discovered. If we therefore conclude that the generalization is true, that every S is Q, we are employing that type of induction called by Bacon *inductio per enumerationem simplicem.* It is extremely likely to mislead, as Bacon pointed out. Even if we examine a very large number of instances, we do not prove our generalization unless we have taken care that any negative instances that do exist shall be brought to our notice. There may be S's which are not Q, and before we can say that "All S's are Q," we must feel sure that such contradictory instances, if they exist, could not have escaped our notice. We must take our S's from different parts of the world, under different circumstances, and so on. The adult influenza patients may have yellow tongues, but not the young ones. The patients in London may have that symptom but not those in the country.

There are, of course, degrees of probability in assertions made on a small foundation. The case of abstract objects, like the circles and triangles of the geometer has been already mentioned. In the case of natural objects belonging to a well-defined class, we know that great similarity exists. What is true of several samples of carefully distilled water, or of

several specimens of the common frog, is likely to be true of all.

This is what we mean by the uniformity of Nature ; which is the ground on which our right to generalize is based. (See above, Chap. XIV. § 1).

Caeteris paribus, the more exactly and distinctly the class S has been defined, the safer will our generalization be. A statement which we should not be able to make with regard to all water we can make more safely of distilled water ; and still more safely about water distilled by some given process and kept in a certain kind of vessel, at a certain temperature. A statement which cannot be made of all trout may perhaps be made with safety about trout of a certain age, living in a certain river, during a given season.

§ 5. Inductions only probable.

Since we can never be sure with regard to natural objects, first, that our definition of a class is sufficiently exact, and, secondly, that even within that class absolute uniformity reigns, no real inductions are quite certain. Putting aside the unimportant class of purely formal inductions (Chap. XIII., § 2), we may say that no inductions can give us more than a high degree of probability.

The propositions of physical science, however carefully tested, are at best only *plurative* propositions. We may be sure that *most* A's are B's, but we can never be certain that all A's are B's. Exceptional

cases may turn up at any time, and only the most reckless of scientific men will claim absolute certainty for the vast majority of the laws they at present accept.

Thus no universal proposition regarding the coexistence or sequence of phenomena, which rests on experience, can be regarded as absolutely certain. No statement that "every X is Y" where the relation between X and Y is not guaranteed by the meaning of the terms—is not, in other words, purely formal—is more than probably true; unless every case of X has been examined. The probability may be very small or very great, but it never becomes certainty. In formal inferences, given the truth of the premises, the result is absolutely certain—always supposing that the process of inference is correctly performed.

At the same time it is most important to remember that for all practical purposes a well-tested induction may be regarded as practically certain. Where we can act freely and without misgiving on the assumption that "X is Y," without any hint as to the possibility of an exception occurring to us, however wide our experience, we have what is as good as certainty. The element of doubt is infinitesimal; and it is only for purely theoretic purposes that we describe the proposition "X is Y" as merely a probable one.

For theoretic purposes the uncertainty, however, still exists. We cannot say that even the Law of Gravitation, or the Laws of Motion are so completely established that an exception is impossible. *Prima facie* an explanation contradicting one of these wide

uniformities, which have survived the tests of centuries of careful observation and experiment, is a bad explanation, and involves an absurdity. But "cocksureness," although a favourite attitude with those scientific men whose minds have not been disciplined by philosophy, is out of place even here. The probability in favour of—say, the Law of Gravitation—may be so great as not to make it worth while for some individual scientific man, who has plenty of other work to do, to investigate the alleged exceptions. But the probability is never so great that we can say that time spent on the inquiry is absolutely wasted.

CHAPTER XVII.

HYPOTHESIS.

§ 1. The Use of Hypothesis.

REFLECTION on the actual process by which we arrive at general truths of a scientific or practical character shows us that we usually start from a small basis of observed fact, and that a *hypothesis* is then formed—that is, a supposition which we know to be at present unproved, but which we think capable of being proved or disproved. This hypothesis takes the form of a general statement, from which we proceed to reason. If "X is Y," then such or such a result will follow. We next proceed to test the truth of the conclusion thus deduced; and if we are satisfied, we consider the hypothesis as proved. When the proof is so thorough that any reversal seems impossible, a hypothesis is often called a *theory*.

"The discovery of a universal law, is always a guess on the part of the imagination made possible by a knowledge of facts. This knowledge is recalled to our memory by the resemblance of the given case to analogous earlier cases" (Lotze, "Logic," § 269).

This is one of the chief services which the imagination renders in science.

The actual method of discovery pursued in science, and to a large extent pursued in practical life, commonly consists of four steps:

(1) *Preliminary observation.*—Only a few instances are usually taken, and these are not as a rule very systematically treated. The use of tables of instances, of statistics, of curves, and so on, is serviceable however, in suggesting hypotheses.

(2) *The formation of a hypothesis.*—The chief problem is the selection of what Whewell calls "an appropriate conception," the grouping of the facts together under some common term. Mill seems to have overlooked the importance and difficulty of this step, and to have thought that the extension of our judgment to fresh cases, the universalizing of the proposition, was the most difficult part of the business.

(3) *Deductive reasoning* from the hypothesis to facts which will be found to exist if the hypothesis be correct. This may take the form of ordinary deduction; or the form of mathematical reasoning, in which quantitative terms are employed.

(4) *Verification.*—Some of the results of this deductive procedure are compared with fact by means of fresh observations and experiments. This leads to the confirmation of the hypothesis, its rejection, or its modification.

An experiment which in itself is adequate to decide the truth of a hypothesis, is called by Bacon an

experimentum crucis; crux being the Latin word for a direction post put at the crossing of roads. A famous *experimentum crucis* was that of Pascal, who caused a barometer to be taken up the Puy de Dôme, and thus definitely negatived the "abhorrence of a vacuum" theory.

The process of Induction, as actually pursued, thus turns out to be a process of *inverse deduction.* Given the conclusion, that "S is P," we have to discover the premises, one or both, which will yield us this conclusion.

§ 2. Kinds of Hypotheses.

Hypotheses vary as to their subject-matter, and as to the use they are intended to serve.

As to their subject-matter, Mill divides them into two classes: (1) Those in which the *law* of action for a cause is assumed, the cause being known to exist. When the movement of the vanes of a radiometer was known to be due to the action of the rays of light falling on it, the *manner* in which these produced their effect was still open to question, and different hypotheses were put forward to explain it.

(2) Those in which the *cause*, or agent, is assumed, the law according to which it was supposed to act being some known law. When a photographic plate is submitted to the action of rays of light coming through a large telescope, properly moved so as to counteract the movement of the earth, many specks are found on it

which cannot be referred to any stars visible even in the strongest telescope. A hypothesis of the highest probability refers these specks to the action of rays proceeding from stars so faint that they can never became visible to the naked eye. Here a cause (rays of invisible light) is supposed to exist, and to act in accordance with the ascertained laws of photographic action.

(3) We may add another to Mill's two kinds. A hypothesis may be one which assumes the co-existence, or collocation, of two or more facts which are not known to co-exist in this particular case.

Mill, it may be here remarked, apparently regards the employment of hypothesis as to some extent abnormal. His language is not very consistent. In one part of his chapter on Hypothesis ("Logic," III. xiv. § 5), he allows that hypotheses "are necessary steps in the progress to something more certain; and nearly everything which is now theory was once hypothesis." But, on the whole, he seems to think that the normal method of discovery begins with a direct "induction to ascertain the law," presumably on the lines of one of his Four Methods explained below—and he speaks with contempt of Dr. Whewell's account of induction, in which the importance of hypothesis was recognized. He puts the terms "induction" and "hypothesis" in marked antithesis. He unduly narrows the field of hypothesis, and restricts the use of the term, so as to exclude from it such an obvious instance as the nebular hypo-

thesis of Laplace ; and he lays down unnecessarily stringent conditions, not only of proof, but of general permissibility.

§ 3. Permissible Hypotheses.

What hypotheses are permissible, and what are not? The usual answer is as follows:

(1) A permissible hypothesis (or, as some writers would say, a valid hypothesis) must not conflict with facts already ascertained, or with laws already proved true.

However, since certainty with regard to laws of nature is only relative, we cannot say that a hypothesis which apparently conflicts with any observed uniformity is necessarily unworthy of investigation. Other things equal, a hypothesis which is in apparent contradiction with well-ascertained laws, is less worthy of attention than one which is not. But there is always the possibility, (i) that the well-ascertained law, though true in the main, is liable to counteraction in particular cases not yet noted; and (ii) that the apparent conflict between the new hypothesis and the old law can be shown, after further observation and discussion, not to exist.

(2) A permissible hypothesis must be capable of leading to deductive reasoning, and the results of such reasoning must be capable of verification.

(3) When the hypothesis belongs to the second kind mentioned in § 2, and supposes the existence of a

given cause acting in accordance with known laws, the cause or agent assumed to exist must be a *vera causa.*

To the expression *vera causa,* used by Newton, no very definite meaning can be assigned. The condition is taken to mean that the cause must be (*a*) one already known to exist; or (*b*) one capable of being known; or (*c*) one whose existence involves no self-contradiction, that is, which is formally possible, because contradicting no law of thought. (*a*) Newton seems to have attached the first of these meanings to his expression. But this is certainly too stringent a qualification; if we insist on it, any hypothesis which involves the existence of a fresh planet or a fresh element hitherto unknown, would be ruled out of court. (*b*) The second meaning is supported by Mill; but it is not entirely satisfactory. Any cause which can be imagined, is capable of being known, if it does exist, under *some* conditions, though not perhaps those we have at our command. If we require that it should be capable of being known under our present conditions of knowledge, we come back to the same stringent conditions as the first. (*c*) The third meaning may be accepted, though it involves a further change in the connotation of the term. It is equivalent to Lotze's requirement that a hypothesis may not legitimately involve anything which cannot be represented to the mind. We may not suppose a circular square to exist, but we may suppose atoms, ether, etc., because so far as our present know-

ledge goes there is nothing self-contradictory in these suppositions ("Logic," § 277).

(4) Other things equal, a hypothesis which does not require auxiliary hypotheses, is to be preferred to one that does. A good hypothesis should be independent, as well as consistent and complete. One which requires bolstering up with a number of other unproved suppositions, is never satisfactory.

§ 4. Verification.

The demand for the most thorough verification of hypotheses which are to be received as laws, is the chief feature which distinguishes modern scientific method from ancient.

The seventeenth century thinkers wrongly ascribed the failure of mediæval science to the use of hypotheses. Newton boasted, "Hypotheses non fingo," and yet his discovery of the Law of Gravitation was a clear case of the use of a hypothesis properly verified.

By verification we mean showing that the hypothesis gives deductive results which are in agreement with fact. "Agreement with fact," says Jevons, "is the sole and sufficient test of a true hypothesis" ("Principles of Science," p. 510). The more facts with which it can be shown to be in agreement, that is the more facts for which the assumed law or cause will account, and the fewer with which it is in apparent disagreement, the more credence will be placed in it.

If we are certain of the fact, "a single absolute

conflict between fact and hypothesis is fatal to the hypothesis," as Jevons says. But what we call a fact of observation is, it must always be remembered, to a large extent a partly unconscious inference, depending tacitly on certain premises. It is usually a generalization which is itself not absolutely certain. And the real conflict may be between the new hypothesis and an old one which has never been properly tested. Thus Newton laid aside his hypothesis of universal gravitation for many years, because he found that it came in conflict with some observations founded on a false estimate then in use as to the radius of the earth.

A hypothesis which is regarded as proved is sometimes called a *theory*.[1] But as proof is, strictly speaking, only relative, we never get more than a very high degree of probability. Thus we cannot draw an absolute distinction between hypothesis and theory. As verification proceeds, and more and more facts are found explicable by the hypothesis, while more and more of the apparent discrepancies with other facts and with received theories are explained away, the hypothesis gradually becomes known as a theory. Thus Darwin's hypothesis of Natural Selection is commonly spoken of as the Darwinian theory ; and no

[1] The word "theory" is ambiguous. It is used for (*a*) an unproved hypothesis—a use to be avoided; (*b*) a proved hypothesis of any kind; (*c*) a proved hypothesis of a complex kind—a whole system of ascertained knowledge, such as the theory of the tides.

one would think now of speaking of the Newtonian *hypothesis* of universal gravitation.

§ 5. Subordinate Uses of Hypotheses.

Hypotheses serve :

(1) To give us general propositions, which must be afterwards verified, as already explained. These hypotheses are intended to be proved and converted into theories, and are put forward in that hope.

(2) As a guide to observation and experiment. Such are put forward as mere *working hypotheses,* without any great hope of their proving correct, but that we may have some kind of guidance in dealing with our accumulated results of observation. A man confronted with a huge mass of facts, for instance with whole volumes of statistics, must make a beginning somewhere. Anything is better than mere haphazard plunging, so he assumes temporarily that a certain relation obtains, that any case of A is a case of B ; and holding this likely, he begins to try to verify it. If any other course strikes him as more likely, he abandons his working hypothesis. When we try to find an object, we follow such a course. We must begin somewhere, and so we assume that the table before us is the best place to begin with ; but if we do not see the object, we form another guess and try the cupboard or the floor.

These working hypotheses, then, differ from the others in being put forward without any great ex-

pectation of their being true, as a guide to observation and experiment. And we may connect their use with the same psychological truth to which allusion was made in the last chapter but one. If observation is to be effective there must be preadjustment of attention, or anticipatory imagination.

(3) There are other hypotheses which do not seem to be put forward with any distinct idea of proving them. Classification involves inference, conscious or unconscious; and is itself necessary for fresh inferences. When we classify or describe phenomena which are tolerably well understood, the element of inference is not very prominent. If I identify a given building as a church, or a number of birds as pigeons, there seems little that can be called guess-work. Sub-conscious inference is present, however, and there is always a leap forwards from the actual sense-data, guaranteed more or less by our previous knowledge. In cases where it is impossible to prove that our inference is correct, the classification or description remains hypothetical. Unexplained facts are grouped together, and a common name is given for the sake of reference. The expression "electrical fluid" is such a *descriptive hypothesis*. "When a phenomenon is of an unusual kind we cannot even speak of it without using some analogy. Every word implies some resemblance between the thing to which it is applied and some other thing" (Jevons).

CHAPTER XVIII.

MILL'S METHODS OF EXPERIMENTAL INQUIRY.

§ 1. The Method of Agreement.

MILL devotes an important part of his treatment of Induction, to a consideration of what he calls the "methods of experimental inquiry."[1] These he seems to regard as the methods by which are ordinarily discovered the uniformities of phenomena which we call natural laws. Whether they are used as Mill thought they were used is open to question, and something will be said at the end of the chapter on the subject. But there is no doubt that the methods laid down by Mill, and before him by Bacon and other writers, are useful for the purposes of *suggesting* hypotheses, and of *testing* hypotheses already formed.

The first method is that of Agreement. We examine a number of cases in which a given phenomenon, A, occurs, and find it in every case accompanied by another phenomenon, X, while all the other accom-

[1] In this phrase there is apparently no reference to the more precise use of the term "experiment" in antithesis to "observation." By experimental inquiry Mill means here inquiry based on experience.

panying circumstances differ, more or less. We are entitled to assume that A and X are causally connected. Mill states what he calls the "canon" of the Method of Agreement in these words:

"If two or more instances of the phenomenon under investigation have only one circumstance in common, the circumstance in which alone all the instances agree, is the cause (or effect) of the given phenomenon."

The basis of this may be expressed in some words of Hume which Mill perhaps had in his mind: "Where several different objects produce the same effect, it must be by means of the quality which we discover to be common amongst them."[1]

The Method of Agreement when employed loosely is practically the same thing as what Bacon calls "inductio vulgaris," or "inductio per simplicem enumerationem." We notice the circumstance X, which always precedes or accompanies the circumstance A, and infer that X is the cause (or part of the cause) of A. It differs from "inductio per enumerationem simplicem" only in the thoroughness with which the process is performed, and the care that is taken to select instances; so that if contrary instances occur, in which A occurs without X, they may be noticed and taken into account.

As a means of suggestion such a method is of the greatest value. Running our eye over a table of

[1] "Treatise of Human Nature," bk. i., pt. iii., sec. xv. Hume also gives in a loose form the foundation principles of the methods of Difference and of Concomitant Variations.

instances, we notice that A is frequently preceded by X, and we form the hypothesis that X is the cause of A. We continue to employ the same process, but now as a test of our newly-formed hypothesis. It is no longer a method of suggestion, but one of verification. If we come across a contrary instance we shall have to explain it away (show that it is not really an exception) or give up our hypothesis, or modify our hypothesis in some way. Or having used the Method of Agreement to suggest a hypothesis, we may lay it aside, and turn to some other method to test the guess which we have made.

The Method of Agreement has been symbolized thus. Let us use capital letters for antecedent or accompanying circumstances, and small letters for phenomena which we regard as consequent on them. Then if we have a number of instances presenting the antecedents A B C, A D E, A F G, A H K, etc., and the consequents, a b c, a d e, a f g, a h k, etc., we know that A is the cause of a. But this way of symbolizing the process is open to very grave objection. It suggests that one antecedent can be ordinarily assigned to one consequent; that A is the cause of a, and B of b, etc. As a matter of fact no such simple relation exists.[1] The whole group of antecedents, A B C, is the cause of the whole group a b c; and we cannot couple A with a, B with b, C with c. It would be better to symbolize thus:

[1] See Chap. XIV. § 3.

X B C D E,	A B′ C D′ R,
X F G H J,	A F′ G′ S T,
X K L M N,	A K L M U,
X O P Q Z,	A O′ P′ W Y,
etc.	etc.

Here the same letters sometimes appear in both columns to indicate that some of the circumstances remain to all intents and purposes unchanged; others appear in the second column with (′) affixed to indicate that they have undergone a change but are still recognizable. Finally some fresh circumstances have appeared, and have taken the place of others. In all this flux of phenomena, however, when we have A as a consequent we have X as an antecedent. We therefore assume that for the phenomenon A to occur in a group of phenomena, X must also be present as an antecedent.

§ 2. Method of Agreement and Plurality of Causes.

Regarded as a method of proof, it is subject to what Mill calls a "characteristic imperfection." The phenomenon A may be due to X in the cases we have examined, not in others. In the second instance it may have been due to H; that is, it would have occurred even if X had been absent as long as H was present; in the third it may have been due to M, and so on. This is the familiar fact which we have already met

with, as, "the plurality of causes," or "vicariousness of causes."[1]

The more cases we examine the less likely are we to overlook this. If we take a sufficiently large number, we are likely to come across some in which X will be absent while A is present. This will at once suggest that X is not absolutely necessary to the production of A. But if the connection between X and A occur more frequently than the laws of probability will account for, we must assume that X is not a mere casual antecedent. It is, probably, at any rate *sometimes* necessary. We cannot however assume this without further proof; and our use of the Method of Agreement is almost necessarily tentative and preparatory.

The result, therefore, of the existence of plurality of causes is that the method can practically only be used as a method of suggestion. "The conclusions which it yields when the number of instances compared is small, are of no real value, except as in the character of suggestions they may lead either to experiments bringing them to the test of the Method of Difference, or to reasoning which may explain and verify them deductively" (Mill). Even if the number of instances be very large, however, the method will give us no very certain results.[2] As a method of proof or verification, it can only be used provisionally. It will serve as a kind of coarse sieve, and will somewhat narrow down possible

[1] See Chap. XIV., § 3.

[2] See Dr. Venn's discussion, "Empirical Logic," chap. xvii. pp. 421-6.

hypotheses. It will select for us a certain number of antecedents as probably concerned in the production of A, but not enable us to pin our faith on one.

There is another objection to the method, arising from the fact that we can never be sure that we know all the antecedents in our instances. The groups of phenomena X B C D E, X F G H J, etc., may be all accompanied by some unnoticed phenomenon, which may be an essential circumstance if A is to be produced, while X and the other phenomena may be none of them absolutely necessary. The presence of a specific microbe has doubtless been the real cause of many diseases which have been put down to changes of temperature, indigestible food, wounds, etc. But either the means of detection at hand were not equal to the discovery of this necessary antecedent, or the attention of observers had not been directed to it.

§ 3. Method of Difference.

If we can obtain two instances, in one of which a given phenomenon A occurs, while in the other A does not occur, no other difference being noticeable ; and if we can detect some other phenomenon X which is present when A is present, and absent when A is absent, while its presence is always accompanied by the presence of A, this phenomenon X is causally connected with A.

Mill's "canon" of the Method of Difference is expressed thus:—"If an instance in which the phenomenon under investigation [designated by A, above] occurs, and an instance in which it does not occur, have

every circumstance in common save one, that one occurring only in the former; the circumstance in which alone the two instances differ, is the effect, or the cause, or an indispensable part of the cause of the phenomenon."[1]

Like the Method of Agreement, this supposes a simplicity which is not found in Nature. But it avoids some of the difficulties which interfere with the use of the first method. It does not try to determine all the conditions; two instances are taken which are practically alike, except in one particular, though we may not be able to assign all the phenomena. Suppose that A B C D E be one of them, and B C D E the other; there may be a large number of other phenomena, F G H, etc., common to both, but these do not concern us, because we take the utmost pains to insure that, whatever they are, they are present in both instances. In order that the two instances may be as nearly alike as possible, it is necessary that we should "take one and the same thing, or event, as nearly as possible at two consecutive instants. There is really no other way open to us; for by supposition, we do not know all the antecedents, and therefore we cannot certainly secure them by the most painstaking selection."[2]

Thus let B C D E represent a galvanometer on a

[1] We cannot, properly speaking, distinguish between "the cause" and "an indispensable part of the cause." No single circumstance can in any strict sense be "the cause."

[2] Venn, "Empirical Logic," p. 414.

certain table in a certain room, at a certain time; and let A B C D E represent everything exactly the same, except that the needle is deflected. What we have to find out is what fresh antecedent or concomitant phenomenon has appeared. Say it is a large key which I have placed on the table. This is again removed, and the needle swings back to its old position. A few repetitions of the experiment assure us that the presence of the key on the table is the only circumstance of importance which has varied. We therefore have the right to assume that the presence of the key is causally connected with the deflection of the needle; either as an essential part of the cause or as part of the effect. We know that it is not the latter; therefore it is the former.

But a little reflection shows that it is, in strictness, impossible to be assured that two instances differ only in the presence and absence of a single attribute, A. When the key was put down on the table many small changes occurred; movements of the body caused vibrations in the floor and in the air; minute optical, electrical, thermal, and physiological changes took place, which we have not reckoned with. We may assume that these unestimated phenomena did not cause the deflection of the needle; but this is only an assumption, an inference resting on grounds more or less satisfactory. Nature is so complex, and our knowledge so limited, that it is always possible that some phenomenon of importance has been overlooked. A large number of repetitions of an experiment, under

varying conditions, help to diminish the uncertainty; but it is never entirely removed.

This method is obviously mainly serviceable for the purpose of verification. It will only be by accident that it suggests a possible cause (as at first in the instance of the galvanometer); but when we suspect a cause we can add just this one new fact to our instances, and see whether the effect which it is supposed to produce will actually be produced. This is how we employed the method in the second and further repetitions of the experiment with the key. All the circumstances remaining otherwise the same in our group of antecedents, we introduced the suspected cause (presence of the key on the table), and saw that the anticipated result followed.

The Method of Difference is, therefore, only applicable under artificial conditions. It is essentially a method of experimental verification. It is employed to determine whether the hypothetical cause really produces the effect which we presume to be due to it. Where experiment is impossible, other methods, *e.g.*, that of Agreement, have to be resorted to.

Mill says it is "perfectly rigorous in its proof" of causation. The method, however, only proves that, in this given instance before us, X is a necessary condition for the production of A; it does not prove that A is always produced when X is present, nor that X is always necessary to the production of A. In other instances, perhaps, Y may be substituted for X with the same results.

§ 4. The Joint Method.

What Mill calls the Joint Method of Agreement and Difference is, in reality, a double employment of the Method of Agreement, positive and negative.

Mill's canon runs thus: "If two or more instances in which the phenomenon occurs have only one circumstance in common, while two or more instances in which it does not occur have nothing in common save the absence of that circumstance; the circumstance in which alone the two sets of instances [ ] differ, is the effect, or the cause, or an indispensable part of the cause, of the phenomena."

Jevons wished to insert "always or invariably" in the place where we have printed a pair of square brackets; since Mill's canon, without this addition, would be satisfied with the following instances:

A B C	a b c	B C	b c
A D E	a d e	D E	d e
A F G	a f g	F G	f g

When obviously the two pairs, A B C, B C with their consequents, would enable us to at once apply the Method of Difference.

The following instances would satisfy the canon, as amended by Jevons:

A B C	a b c	P Q	p q
A D E	a d e	R S	r s
A F G	a f g	T W	t w

We collect instances of the presence of the phe-

nomenon under investigation, and instances of its absence. If we are seeking the cause of a, then we take instances in which a is present, and find that among their immediate antecedents or concomitants A is always present, and that where a is absent, A is absent from among the antecedents and concomitants. Or if we already suspect A is the cause of a, we can use the method as one of verification; for taking instances in which A occurs as an antecedent or concomitant, we find a always present among the consequents, and when A is absent, a is also absent.

It should be noticed, however, that the negative instances add very little to the efficiency of the method, unless they are cognate to the positive instances. They must be cases "as much resembling them [the positive] as possible," as Mill himself remarks ("Logic," Bk. III. ch. ix. § 1). If we are seeking a cause for the phenomenon of animal heat, it is of little use to select instances of the absence of heat from the mineral, or even the vegetable kingdom. These help us very slightly. But negative instances taken from the animal kingdom throw considerable light on the problem. The more analogous our negative instances are to our positive, the more nearly does the method approach in validity to the direct Method of Difference.

Our formulæ must be something of this kind:

A B C	a b c	B′ C′	b′ c′
A D E	a d e	D′ E′	d′ e′
A F G	a f g	F′ G′	f′ g′

Chemistry gives us opportunities for getting analogous instances. Various specimens of a substance, if pure, differ so little that B C and B′ C′ are almost exactly the same; and the presence and absence of the A may be regarded as the only difference. So that the method passes over into that of Difference. In Sociology, on the other hand, we never get two cases so much alike that we cannot discern many other disparities, as well as the presence and absence of A.

§ 5. Method of Residues.

The three methods hitherto noticed have been all methods of *elimination*. "The Method of Agreement stands on the ground that whatever can be eliminated, is not connected with the phenomenon by any law. The Method of Difference has for its foundation, that whatever cannot be eliminated, is connected with the phenomenon by a law."[1] And the Joint Method obviously stands on the same basis.

The fourth method formulated by Mill, that of Residues, is a modification of the Method of Difference. From the known effects of A and B separately, we infer their conjoint effect, and subtract this (a b) from the total effect to be investigated, a b c. The remaining part of the effect, c, is due to any further antecedent, C, which we may discover.

[1] Mill, "Logic," III. viii. § 3.

Mill's canon runs thus: "Subduct from any phenomenon such part as is known by previous inductions to be the effect of certain antecedents, and the residue of the phenomenon is the effect of the remaining antecedents."

Jevons cites as a simple instance the ascertaining of the exact weight of any commodity in a cart, by weighing the cart and load, and then subtracting the known weight of the cart. The method usually requires quantitative data, as in this case; but it can be applied qualitatively. Certain unpleasant symptoms (c) noticed after dinner, must be due to a special dish (C), because the feelings (a b) produced by other dishes (A B) are well known, and can be mentally subtracted, or as Mill calls it, subducted, from the total effect.

A little consideration will show that the Method of Residues is not very conclusive as a method of proof. We can never be certain that C is the only residual antecedent; some difference in temperature, in our manner of spending the day, or in our general health, may really have caused the unpleasant feelings which we have placed to the credit of the suspected dish.

But the method is valuable as a means of suggestion. The study of *residual phenomena* is a most fertile source of great discoveries. The discovery of the planet Neptune, by Adams and Leverrier in 1846, was due to it. In the year 1894 it led to the discovery of a new constituent of our atmosphere, argon. Supplemented by deductive verification, which shows, *à*

priori, that C may be a cause of c, or by experimental verification by the Method of Difference, the method is of the greatest value.

§ 6. Method of Concomitant Variations.

This method is another special application of the Method of Difference. If we have a set of antecedents, A B C D, even if A cannot be entirely removed, it may be modified in amount or in some other way. If a corresponding change occurs in the group of consequents a b c d, so that one of them, a, becomes modified in amount, or in some other way, we assume that there is some causal connection between A and a. In cases where a complete vacuum cannot be produced, we may often argue with great security that the presence of air (or some other fluid) is a necessary condition of certain phenomena—because, as the vacuum becomes more and more perfect, the phenomena diminish in amount.

Mill's canon is stated in these words:—"Whatever phenomenon varies in any manner whenever another phenomenon varies in some particular manner, is either a cause or an effect of that phenomenon, or is connected with it through some fact of causation."

The Method of Concomitant Variations can be applied when the Method of Difference cannot. It is thus specially valuable to determine the effects of causes which are always present to some extent, though in very various degrees—such as gravity and

heat. It can be easily applied when the cause varies in quantity; it is thus specially serviceable when we have exact quantitative data. As Professor Bain points out, it is not liable to be frustrated by intermixture of effects as most of the other methods are. "If a cause happens to vary alone, the effect will also vary alone, and cause and effect may be thus singled out under the greatest complications. Thus, when the appetite for food increases with the cold, we have a strong evidence of connection between these two facts, although other circumstances may operate in the same direction" (Bain, "Induction," III. viii. § 6).

It must be remembered, (1) that the law of variation which the quantities seem to follow within the range of our observation may not hold outside it. Change of quantity may, after a time, produce change of kind; what Mill calls a *heteropathic effect* may be produced. Friction carried beyond a certain point may produce an explosion. Pleasant stimulation if steadily increased, passes, after a while, into pain. And (2) different laws of variation may produce results which, within narrow limits, shall be practically identical.

§ 7. Mill's Treatment of the Methods.

The exact position which Mill intends to assign to his "methods of experimental inquiry," or "methods of induction," is not quite clear.

The ordinary means by which we arrive at the laws which are exhibited by "complex effects" (and we

have seen that all phenomena are complex effects[1]) he calls the Complete Deductive Method. It consists of three steps: (1) Induction, by means of the "methods of experimental inquiry"; (2) Ratiocination, a calculation from these known laws what will be the effect in any given combination of these causes; (3) Verification, by direct observation, or by new inductions, or by deduction. This statement of the Complete Deductive Method bears some resemblance to the account given in Chap. XVII. § 1 above. But it will be noticed that Mill regards his first step as in itself a complete induction, and as giving us a result which is thereby known to be true. The third step, which he calls verification, is not a proof of the truth of the general law, but of the results of the deductive process. He regards the "methods of experimental inquiry" as giving results which are in themselves certain, and which do not require verification.

Indeed, he expressly draws a distinction between this Direct Complete Deductive Method, and the Inverse Complete Deductive Method, where the formation of a Hypothesis is substituted for the direct use of the experimental methods. It is true that he occasionally speaks of Hypothesis as a possible part of the ordinary method of discovery, and in some places he speaks of his Four Methods of experimental inquiry as though they were "tests for induction similar to the syllogistic test of ratiocination." But on the

[1] See Chap. XIV. § 4, above.

whole he seems to think that they are the normal and ordinary method of arriving at general propositions after observation of phenomena. The most important statement on the other side, which if it stood alone would be quite conclusive, and which gives a much truer view of the real value of the method, is that at the end of Bk. III., chap. x. § 3. He points out here that, owing to the interference of causes with one another, and the intermixture of effects—a case which he recognizes, though not with adequate emphasis—the four methods of inductive investigation are of secondary importance. "The instrument of Deduction," he says, "alone is adequate to unravel the complexities proceeding from this source; and the four methods have little more in their power than to supply premises for, and a verification of, our deductions."

If this be Mill's final view, which is open to doubt, it comes to much the same as that advocated here that the famous Methods are of value:

(1) To suggest hypotheses;

(2) As a means of verification of hypotheses.

§ 8. How the Methods are actually employed.

Let us take a case which actually occurred. In August, 1883, an epidemic of typhoid fever occurred in Camden Town. The medical officer, Mr. Shirley Murphy, prepared a plan of the district, on which he marked all the houses which had been attacked. His

scientific knowledge at once suggested to him a number of hypotheses as to the origin of the attack. Putting them briefly, they were (1) the Regent's Canal; (2) the water supply; (3) the sanitary arrangements in the houses; (4) the milk supply. The use of his plan, and inquiry at the houses, showed that the first three hypotheses were invalid. The houses attacked were not usually near the canal. Two water companies supplied the district, and houses supplied by both companies were attacked with impartiality. Sanitary defects existed both in attacked houses, and in those which escaped the disease; while the sanitary arrangements in some of the attacked houses were perfect. So far the positive and negative Methods of Agreement (Joint Method) had been applied as a means of testing the hypotheses arrived at,—not, be it noticed, to suggest the hypotheses, which were largely due to deductive reasoning from general laws of hygiene.

The fourth hypothesis remained. By the application of the Method of Residues, the case in favour of this fourth hypothesis was not *proved* (as Mill suggests) but perceptibly strengthened. Now, by a direct use of the Method of Agreement, "it was discovered that out of 431 persons attacked, 368 were definitely known to obtain their milk from one particular milkman, Mr. X., while the remaining 63 might well have indirectly obtained it from him also. . . . . Out of all the houses attacked, 78 per cent. received their milk from Mr. X." This use of the Method of Agreement was, however, probably twofold. After the first score

or so of cases a strong suspicion was probably aroused in Mr. Murphy's mind that Mr. X.'s milk was the true *causa sine quâ non;* while the subsequent cases were, it would appear, rather used by way of test, or verification, of the hypothesis so formed. Whether this was so, the report[1] gives no information.

Mr. Murphy now definitely verified his hypothesis by examining the shop and premises of Mr. X., and the farms from which Mr. X. obtained the milk. He succeeded in proving that one of these farms, near St. Albans, was infected with typhoid, and thus finally showed the adequacy and truth of his hypothesis. As the older logicians would have said, he proved that the typhoid fever infection in Mr. X.'s milk was a *vera causa.*

[1] All the details of Mr. Murphy's admirable investigation I have taken from the "Pall Mall Gazette," of October 15th, 1883.

CHAPTER XIX.

EXPLANATION.

§ 1. Explanation.

THE object of explanation is to show that some given fact, or given law, is not exceptional, but that it can be brought under some law or laws already known to be true. It is our answer to the question, "Why does this occur in this way?" It relates a fact hitherto isolated in experience, to other facts.

Scientific explanation does not differ materially from popular explanation. It is, however, more precise and better verified. And it differs from popular explanation in offering laws which are simpler in themselves and of wider application, though they may be less familiar, while popular explanation hesitates to do this. The scientific man explains by reference to things which are in the phrase of the mediæval philosophers "*notiora naturæ*," and not "*notiora nobis*." The law of universal gravitation is simpler and more abstract, and thus wider in scope, than the law that tea-cups left unsupported fall to the ground; but it is less familiar.

A *fact* is explained by being brought under some law. It is shown to be a particular case of some

uniformity. Why does this phenomenon occur here now? Because it is a case of a certain kind, which comes under a particular law. Why do I observe certain movements in the cilia of a rotifer under the microscope? The answer may be, that all rotifera present these appearances; or it may go deeper, and suggest some causal relation between these movements and the structure of the animals. But in either case the fact is brought under a general law.

A general law, however, is only one degree less isolated than a single fact. We are driven to bring such a law under laws still more general.

Mill enumerates three kinds of explanations which may be applied to laws.

(1) The law that "A is B" may be resolved into the laws that "A is X," and "A is Y," and the law that where "X and Y are conjoined, B is produced." Thus, to use Mill's example, the orbit of a planet is shown to be due to the fact that the planet is acted upon by two forces, the tangential impulse and the force of gravity, and that the joint effect of these two is movement in a closed curve. Each of these laws is simpler and less specialized than the law that the planet moves in such an orbit.

Here, as Dr. Venn insists,[1] there is something more than mere generalization. There is analysis as well.

(2) Detecting an intermediate link in the sequence of causation. The sequence, A is followed by B, is

[1] "Empirical Logic," pp. 498, *sq.*

shown to be a complex case due to the two laws that A produces X, and that X produces B. We never succeed in getting the causal relation so precise, that no intermediate event can possibly be interposed between A and B. The simplest sequence with which we are familiar can usually be analyzed by scientific men into a series of events. Thus the production of a flame when a match is struck resolves itself into several consecutive events: (*a*) friction produces heat of a sufficient intensity, and (*b*) this heat breaks up some of the unstable chemical composition on the head of the match; (*c*) some ingredients of this being liberated unite with explosive energy with the free oxygen of the air; (*d*) this sets fire to all the rest of the composition.

(3) A specific law is shown to be a particular case of a wider law. The explanation that tea-cups fall to the ground because every particle of matter attracts every other, is, in point of fact, a re-statement of the same fact as part of a large group of facts. It does not attempt to break them up and analyze them; but merely generalizes, and shows that the case of tea-cups is included in a uniformity of incalculably wider reach.

§ 2. Exceptional Phenomena.

A scientific law is only the expression of one observed uniformity. If the uniformity has been inadequately expressed, we have an *exception*. The excep-

tion arises from want of precision in our expression of what we have observed. The preliminary classification has, perhaps, been badly performed, and the subject of our proposition is at fault. We have said "All A is X," when we should have said "All A B is X." If we come across an A C which is not X, we call it an exception to the rule that "every A is X." Or we have imperfectly described the phenomena observed in the instances examined, and said "All A is X," instead of "All A is X′." Here, as long as X′ closely resembles X, we do not find or notice any irregularity. But if the quality X′ happens to differ considerably from X in some special case, we call this case an exception.

In such cases we can see that the exception is an exception to our expression of a uniformity, which we believe we have observed. As a matter of fact, the uniformity does not exist, and our law is only an approximation to the truth. The fault lies with us.

There are no laws in Nature herself. Man makes the laws, as he makes the classes; both are purely subjective; and the laws depend on the classification. Hence all laws are in their essence abstract and hypothetical. If the individual things of which we are speaking completely conform to our definition, if they are true members of the class we have marked off, they will possess the qualities which our "law" declares to be possessed by all members of the class.

Exceptional phenomena are of the greatest value in science. They arouse wonder, and draw attention not

only to themselves but to the laws to which they are exceptions. Psychologists tell us that one of the chief factors in producing efficient attention is novelty; and although what is familiar may be as worthy of notice as what is new and strange, it does not affect us in the same way.

Bacon drew attention to the importance of exceptional phenomena for the purpose of scientific discovery. His *instantiæ monodicæ*, or *instantiæ irregulares*, and his *instantiæ deviantes* are examples. He recognizes not only their stimulating effect on the mind, but also the danger which arises from the disproportionate impression they make, to the exclusion of others equally instructive. By *instantiæ monodicæ* he means irregular kinds, "miracula specierum," while the *instantiæ deviantes* are as it were "errores naturæ," and are "miracula individuorum," or what modern naturalists call "sports."

Jevons, in his "Principles of Science" (chap. xxix.), gives a classification of exceptional phenomena, but it has small value.

§ 3. Extension of Empirical Laws.

Derivative laws, as we have seen, are those uniformities which have been shown to be special cases of more general and certain laws, or are *supposed* to be capable of such affiliation on more general and certain laws. The latter, it will be remembered, are called *empirical laws*.

All such derivative laws, being *ex vi termini* com-

paratively specific and narrow, are more liable to exception than the wider laws. The law, "A follows B," and the law "B follows C," are less liable to exception than the law "A follows C," which is a derivative law depending on both. If an effect requires for its production the presence of several conditions, it is more liable to counteraction than an effect which requires fewer conditions. The flaming of a match when struck is more liable to be hindered than the production of heat by friction, or than the deflagration of the head of the match when sufficient heat is produced. There are two possibilities of frustration instead of only one. The collocation, or co-presence, of *all* the necessary conditions is more difficult to secure than the presence of only those circumstances which produce heat, or the presence of only those circumstances which, when heat exists, produce deflagration.

We can never be quite certain that the conditions necessary for the production of a highly specific effect (and to such effects derivative laws commonly refer) are present. But experience will give us a very high degree of probability that collocations which have been observed to occur will continue to occur within narrow limits of time and space.

How do we know the sun will rise to-morrow? The phenomenon of sunrise depends on certain optical laws, and on the continued existence of certain material objects. The continued existence of a luminous sun and of a revolving earth is implied. The laws of the propagation of light might remain the same; but if sun or

earth disappeared, or changed their nature suddenly, the sun might not rise. That they will remain unchanged is an inference justified by very long experience, and by our ignorance of any cause which might produce the change within twenty-four hours. "The chance, therefore, that the sun may not rise to-morrow amounts to the chance that some cause, which has not manifested itself in the smallest degree during five thousand years, will exist to-morrow in such intensity as to destroy the sun or the earth, the sun's light or the earth's rotation, or to produce an immense disturbance in the effect resulting from these causes."[1]

The extension of an empirical law rests then on the probabilities of the continued uniformity of very complex phenomena. "In proportion, therefore, to our ignorance of the causes on which the empirical law depends, we can be less assured that it will continue to hold good; and the further we look into futurity, the less improbable is it that some one of the causes, whose co-existence gives rise to the derivative uniformity, may be destroyed or counteracted."[2] A cause which is at present too small in amount to arrest attention, may, after a sufficient lapse of time, bring about an exception; and the uniformity may disappear.

Derivative laws, whether resolved or as yet unresolved (empirical laws), can, then, only be extended to cases *adjacent in time*. They cannot be extended to cases *adjacent in place*, unless the new cases to which

[1] Mill, "Logic," III. xix. § 2. [2] *Ib.* § 3.

they are extended are "presumably within the influence of the same individual agents." If we know that the same causes are at work, we may extend the generalization to fresh cases. A special modification of some biological form found in some part of a country is likely to be found in other adjacent parts, where the same causes are at work. And even though we do not *know* that the same causes are acting, we may be able to assume it with considerable probability. Even if we do not know that the same conditions are present in other adjacent places, we may commonly assume that they are. But the narrower the limit of space the more probability there will be in our inference.

Uniformities of mere co-existence are, as already remarked (Chap. XIII. § 3) less certain than those which rest on causation. They are purely empirical laws.

§ 4. Analogy.

It is usual in works on Logic to draw a distinction between what is called the proper use of the term Analogy, and the loose, or popular use. In the former and stricter sense we mean by it a resemblance between the *relations* of things; in the latter sense we mean by it any resemblance between the things themselves, which yet amounts to a great deal less than absolute likeness.

"Analogy," says Kant, "does not signify, as is commonly thought, an imperfect likeness between two

things, but a perfect likeness of relations between two quite dissimilar things." And most other logicians express themselves in a similar way.

When in the early years of this century Pitt was called the "pilot that weathered the storm," we have an analogy even in the stricter sense. The analogy may be expressed in the same way as a mathematical proportion, which is the declaration of the equality between two ratios or relations of number.

Pilot : ship :: Pitt : state.
Pilot : storm :: Pitt : French Revolution.
Ship : storm :: state : French Revolution.

Such an analogy gives rise to metaphor and similes of the most effective kind. As a foundation for argument it has considerable value ; as a foundation for proof very little. It suggests lines of thought; but he who uses it is always open to the objection that the analogy is superficial and fanciful. To prove that the analogy is real and substantial will usually take more trouble than to prove the original point without the assistance of analogy. Thus it is more satisfactory to attack directly the question: "Ought the colonies to assist England by arms against foreign foes?" or "Ought they to contribute to the expenses of imperial defence?" directly, than to try and establish the analogy—

Mother : children :: England : colonies.

The looser type includes any kind of resemblance. The "argument from analogy" very often means

merely the argument from likeness. Resemblance of relation usually involves some degree of resemblance in the things.

Two objects, A and B, resemble each other in several points. They possess in common the qualities v, w, x, etc., and we infer that because A possesses further the quality y, B will also possess it.

Logicians commonly contrast this process with Induction. Generalization, or Induction, involves many objects, A, B, C, D, etc., all possessed of at least two properties, x and y; and we infer that another object, M, which possesses x, will also possess y, on the ground that we have found that in all the other cases x and y went together. Here the number of cases examined is large, while the qualities we are concerned with may be only two in number. The denotation is wide; the connotation may be narrow. On the other hand, in the argument from analogy or resemblance we may have only two objects to compare, and therefore we need more than the one point of agreement, x, in order to infer the presence of another, y, when we do not know that the two are causally connected. Every additional quality will strengthen the argument. The connotation should be large, because the denotation is small.

Mill says that in cases where the resemblance is very great, the dissimilarity very small, and our knowledge of the subject matter extensive, the argument from analogy (resemblance) comes very near a valid induction. This however seems to be an error.

"In all cases," says Lotze, "when we believe we can

prove by analogy, the analogy in fact is distinctly not the ground of the conclusiveness of the proof; it is only the inventive play of thought by which we arrive at the discovery of a sufficient ground of proof" ("Logic," § 214). Analogy is of the greatest value as a means for suggestion. It suggests hypotheses, both inductive and descriptive; and the mind that readily sees analogies is usually fertile in new ideas.

But in no case is analogy a substitute for proof. It can at best only lead to a very low degree of probability; whereas, inductions proper, that is, generalizations arrived at after examination of many instances, even when not regarded as absolutely proved, often have the very highest degree of probability.

The argument for the existence of consciousness in the lower animals is an argument from analogy; and every psychologist feels the inadequate nature of it.

CHAPTER XX.

CLASSIFICATION.

§ 1. Classification and Formal Division.

THE process of classification is involved not only in all processes of judgment and reasoning, but in those of conception and perception. Directly we name an object we implicitly refer it to a class. If we think of an attribute we implicitly divide all things into two classes—those which possess, and those which do not possess, the attribute. Some automatic and half-conscious or unconscious classification is therefore involved in the use of general names.

When by the process of Definition we render clear and distinct the exact connotation of a name, that is, the attributes implied by it, we at the same time render more definite the classes to which it applies.

It is important to understand the relation between the process of Division already described (Chap. VI. §§ 4, 5), and the process of Classification. Logical Division is a purely formal process. Given a class of objects, A, we can point out *à priori*, without reference to any particular subject matter, to what general conditions the classification must conform if it is to be of

any value, theoretical or practical. Whatever A may be, the rules given in Chap. VI. § 5, must be complied with.

In material or applied Logic, we can go farther than this. Taking some notice of the subject matter with which we deal, we can lay down a few general truths as to the kinds of classification best adapted to certain departments of knowledge. No longer assuming that we are dealing with mere symbols, and taking into account the fact that our names and concepts are not absolutely objective—that the world of perception and practice can never be adequately represented by A's and B's—we point out the differences of procedure which will properly be followed in grouping various kinds of things. We have already remarked that even in what is called Formal Division the process is not absolutely formal and *à priori;* since the *fundamentum divisionis* must be an attribute not included in the connotation of the name. But whereas in formal division our *fundamenta divisionis* are assumed to be already given, here we explicitly point out the kinds of attributes best adapted for *fundamenta divisionis.*

Material Logic, then, proposes to lay down general principles for our guidance in the matter of classification. Passing beyond the automatic classification already spoken of as involved in all thought and speech, it aims at showing us how we may best, for the purposes of clear thought and valid inference, arrange the objects with which we have to deal. In the words of

Mill:—"The general problem of Classification in reference to these purposes [viz., those of Inductive Logic] may be stated as follows: To provide that things shall be thought of in such groups, and those groups in such order, as will best conduce to the remembrance and to the ascertainment of their laws."

The importance of classification to induction has already been dwelt on (Chap. XVI. § 3). All valid generalization involves the correctness of the preceding classification.

§ 2. Artificial and Natural Classification.

It is clear that classification is of two different kinds. We may classify objects on lines which are obviously arbitrary and, so to speak, artificial; or we may endeavour to conform to the classes which seem to come from the hand of Nature. If we divide animals into those which are five feet long and upwards, and those which are less than five feet; or into those which have been exhibited in the Zoological Gardens, London, and those which have not, we are dividing in a purely arbitrary way. Our *fundamentum divisionis* is made by ourselves for some special and very limited purpose. Our classification is in no sense a natural one. When a collector is dealing with artificial objects, such as books, book-plates, or postage stamps, several kinds of classification may be equally rational and convenient, because serviceable for equally important ends, and all are equally artificial.

When, however, we come to natural objects, above all to plants and animals, this is not the case. Nature seems to give us our classes ready made. All that we have to do is to try to recognize them. If we do not recognize them, the penalty we pay is incapacity to draw useful and valid inductions. How many general propositions can be affirmed of just those animals in the Zoological Gardens, as opposed to animals not confined there? But of some well-marked "natural class," such as lions or reindeer, a number of general truths can be ascertained—important attributes can be found which will be true of all members of that class.

These "natural classes" are distinguished by an immense number of common attributes which are, at any rate at first sight, more or less independent and unconnected. While we can indicate in a moment the essential points of resemblance between the members of the class "white things" or "heavy things," there is no likelihood of our ever discovering all the attributes common to different specimens of the kinds "sulphur" or "man." *Real Kinds*, or *Natural Kinds*, is the name given by Mill to the classes thus marked off by an indefinite number of common attributes. He apparently conceived of the co-existence of the permanent attributes as uncaused, and inexplicable. Although he lived till a period when the doctrines of Darwin and Wallace and Spencer had been generally accepted by scientific men, he made no alteration in his view.

Real Kinds are for Mill classes marked off by an inexhaustible number of differences, *not referrible to any common cause.* They are "classes between which there is an impassable barrier."

The distinction thus drawn between a Natural and an Artificial classification, between one which best fits natural objects, and one which can only be applied satisfactorily to artificial objects, is less absolute than Mill imagined. To begin with, artificial objects, like books or pictures, which certainly cannot be said to belong to classes "made by Nature" present many, indeed an indefinitely large number of common attributes. Massive scientific treatises have been written on the common characters which are found in particular classes of books, or in paintings attributed to a particular school. Such artificial groups present many of the features which Mill attributes only to natural groups.

Then again, the line cannot be drawn sharply between a classification due to the recognition of one or two attributes only, and a classification due to the recognition of an inexhaustible number of attributes. The attributes of dog or man are not, as Mill thought, independent; but are probably in a very large degree "referrible to a common cause," and "derivable from each other." On the other hand, the presence of one or two attributes in even an artificial object commonly involves the presence of many others. That a coin is issued in France, or that a picture is a seventeenth-century Dutch one, involves

a long train of consequences, more or less independent in appearance, but more or less connected in reality.

§ 3. Special and General Classification.

It is better to substitute for the distinction between Artificial and Natural classifications, the distinction between classification for special purposes and for general purposes.[1] A classification of the former kind aims at fulfilling some quite definite and limited purposes. The aim of classification of names and addresses in a directory is, clearly, that they may be easily found. The classification adopted is therefore alphabetical. No general propositions can be drawn as to the people whose names begin with the same letter. The obvious merit of the arrangement "consists in the extreme celerity with which it isolates the element we are in search of, the alternatives being twenty-four [or rather twenty-six] at every step, and all but one being instantly laid aside." Our aim being to identify an individual and ascertain his address, this is by far the best arrangement. But it yields us no further information; except in so far as the "Mac's and O's" can be grouped together as Celtic names, and the Browns and Smiths as purely English ones. A classification of individuals under their trade would give us materials for interesting speculation as to the popularity or the reverse of certain trades, the localities in which they are carried on, and so forth. This

[1] Venn, "Empirical Logic," chap. xiii.

would approach a "Natural" or general classification, which has for its two chief purposes the easy remembrance of knowledge already obtained, and "aid and stimulus to fresh study."

If a general system of classification is followed, we gain the power of making a "maximum amount of aggregate assertions with a minimum number of propositions." In other words, we can make many true universal propositions about the class. We can make many assertions about the class of brewers, or of painters, or of solicitors, each of which assertions will apply to a large number of individuals.

§ 4. Classification not the Work of Nature.

In the third book of his "Essay concerning Human Understanding," Locke lays down the doctrine that, even in the case of natural objects, classification is the work not of Nature, but of man. The classes "depend on such collections of ideas as men have made; and not on the real nature of things." In a certain sense they are "arbitrary," since they depend on the accidents of special needs and temporary conditions. Objects are, indeed, made by Nature to resemble each other, but the classes are made by the mind of man, which selects such points of resemblance between things as interest it most, and classifies the things in accordance with those interests.

This doctrine is accepted by modern writers. Mill, for instance, speaking of "natural" and "artificial"

classification, says: "The differences [between objects] are made by nature in both cases; while the recognition of those differences as grounds of classification and of naming, is equally, in both cases, the act of man; only in the one case, the ends of language and of classification would be subverted if no notice were taken of the difference, while, in the other case, the necessity of taking notice of it depends on the importance or unimportance of the particular qualities in which the difference happens to consist" ("Logic," Bk. I. Chap. VII. § 4).

Mill, however, did not fully recognize the implications of the statement, or he would hardly have committed himself to the doctrine of Natural Kinds.

All classification is relative to some definite purpose. The anthropologist, the theologian, the psychologist, the artist, the lawyer, classify men in different ways. But no classification is in itself better than any other. There is a growing disposition to assume that the particular classification which suits the purposes of what is called Natural Science, and especially of the sciences connected with Biology, is somehow or other absolutely superior in kind to all others. This, however, is a mistake. The purely speculative interests of the man of science are best served by the special classification he adopts; but those speculative interests have no claim to over-ride all others. For his limited and definite purpose the biologist places man with the higher apes. Man differs from them only in degree; between them and him there is "an all-pervading

similitude of structure," to use the words of Sir Richard Owen. The zoologist classifies by reference to morphological characteristics. And on that ground he is hardly justified in making Man an "Order," side by side with the great and highly varied orders, such as the Insectivora, and the Quadrumana. All this may be granted; while at the same time the psychologist, the theologian, and the moralist, may claim that for *their* purposes—purposes more essential to actual conduct, that is to the real and necessary business of life—we must draw a much broader and deeper distinction between Man and Ape. The biologist can lay no sort of claim to having seen deeper into the purposes of Nature than other men.

When we speak of a *scientific classification*, we mean one specially adapted for the purposes of some one science, or perhaps of several sciences. A satisfactory scientific classification will thus be the "general" kind, already spoken of. It will "classify objects according to the whole of their resemblances and differences, so far as they are recognized by the science in whose service the classification is made" (Fowler, "Inductive Logic," p. 55). At the same time, while it takes care that the properties it selects for *fundamenta divisionis* are (1) marks of other properties, both numerous and important, it will also see that they are themselves (2) notable and important properties, easily observed.

It is impossible to classify natural objects in such a way that all of them shall easily fall into a well-

defined class. The boundaries of species, as Locke said, are not immovable: Nature from time to time presents us with objects that fall into no recognized class. The Law of Continuity is found to be true in all departments of experience; and the isolated cases of a Natural History Museum, as we have already remarked, imply a discreteness which does not exist in the actual world.

§ 5. Classification by Type.

The classes recognized by the scientific man, that is the careful and systematic student of Nature, are suggested by the general and bolder resemblances between objects; but they are revised and rendered definite by drawing up precise lists of the attributes which entitle an object to be placed in a class. In other words, they are suggested by *Type*, and determined by *Definition*.

Dr. Whewell, the famous Master of Trinity College, Cambridge, who was the most powerful critic of Mill's doctrine of Induction, maintained that the classes recognized by science, the Natural Kinds of Mill, are "given by Type and not by Definition."

"The class," says Whewell, "is determined, not by a boundary line without, but by a central point within; not by what it strictly excludes, but by what it eminently includes; by an example, not by a precept; in short, instead of a Definition, we have a Type for our direction." By a Type, he means "an example of

any class, for instance, a species of a genus which is considered as eminently possessing the character of the class." "The type must be connected by many affinities with most of the others of its group ; it must be near the centre of the crowd, and not one of the stragglers." [1]

Mill admits that there is some truth in this doctrine. "Our conception of the class, the image in our minds which is representative of it, is that of a specimen complete in all the characters [which are found in most of the objects in the class, and most of which characters are found in all the objects] ; most naturally a specimen which, by possessing them all in the greatest degree in which they are ever found, is the best fitted to exhibit clearly, and in a marked manner, what they are. It is by a mental reference to this standard, not instead of, but in illustration of, the definition of the class, that we usually and advantageously determine whether any individual or species belongs to the class or not" ("Logic," Bk. IV. Chap. VII. § 4).

And for the higher development of classification which is termed *serial,* Mill recognizes that the assumption of a Type is absolutely necessary.

§ 6. Classification by Series.

The work of classification is only half done if it results merely in the recognition of a number of unconnected

[1] Whewell, "History of Scientific Ideas," ii., 120-122 ; quoted by Mill.

groups. The groups must be arranged under each other in such a way, that those which are most nearly alike are brought nearest each other. Any division which arranges widely different classes together, and separates those which closely resemble one another, is *pro tanto* a bad and useless one.

The obvious resource is to take some salient phenomenon exhibited by all the groups, and to arrange them according to the degree in which they exhibit it. Such an arrangement is called a *serial classification.* It obviously puts the instances into a convenient order for the application of the Method of Concomitant Variations. Thus the general aim of zoological classification may be regarded as the exhibition of the phenomenon of Life, in a series extending from its slightest and simplest form in the Protozoa, to its fullest and most complex form in Man. As already remarked, the assumption of Type Species is indispensable for purposes of serial arrangement.

Where natural objects are dealt with, however, no arrangement of classes can be strictly serial. What we constantly find is an arrangement which requires not one, but two or three dimensions of space to exhibit it; it will require not a *linear* series, but rather a set of lines radiating in as many directions as space possesses. Where only one phenomenon varies regularly, the groups can be placed in a line one below another; where several phenomena vary, this simple arrangement is no longer possible. This Dr. Whewell seems to have seen, for he thought the doctrine of "a

series of organized beings" to be "bad and narrow philosophy." Mill's reply[1] shows that he did not intend us to understand by a *series* one line only. "It would surely be possible to arrange all *places* (for example) in the order of their distance from the North Pole, though there would be not merely a plurality, but a whole circle of places at every single gradation in the scale."

[1] Footnote to "Logic," Bk. IV., Chap. VIII., § 1.

CHAPTER XXI.

SCIENTIFIC LANGUAGE.

§ 1. Language and Thought.

LANGUAGE serves three chief purposes: (1) it serves as a means of communicating our thoughts; (2) it serves as a means of recording our thoughts; (3) it serves as a means of directly assisting thought. Any system of signs, auditory, or visual, or tactual, which answers these purposes, is a language.

It is not necessary to dwell on the first two parts: the third deserves a moment's consideration. Psychology shows us that language is practically essential to thought. Without language general notions, or concepts, could scarcely exist. It is the name which binds together the group of simpler ideas that we call its connotation; and which enables us to recall this group, and to wield it as a whole. In thinking, we substitute the verbal sign for the mass of dimly recalled presentations, and of unpresentable relations of these presentations, which is what we call a concept. If one had always to call up even a faint image of a *dog* when one's thoughts involved a reference to dogs, thinking would be difficult. But what repre-

sentation is there to call up when we think about *fidelity*, or *homogeneity*, or *susceptibility?* Some representation is necessary, and the name serves the purpose as nothing else could. An auditory, or visual, or muscular representation of the name becomes nascent, that is, begins to rise in consciousness; and before we are fully conscious of it, our thought has passed on to something else. But this symbol of a thought has done its work.

In algebra, when we have to deal with long and complex expressions, we often substitute for them some simple symbol, such as the Greek letter Σ. We can now treat Σ as if it were an ordinary term, and use it almost mechanically, knowing all the time that we could, if necessary, substitute for it the whole of the expression for which it stands. This serves as an illustration of the use of the name. We can use it almost mechanically, so long as we know that at any moment we can recall the list of attributes which form its connotation.

Language is thus a direct help to thought. It secures the thought the moment that it arises, if we wish it, in the form of a *proposition*, or in the still more convenient and compact form of a *term*. When we have to think over a matter of difficulty, we find ourselves repeating the premises and the conclusion to ourselves in unuttered language. With some men this mental language passes into the muscular movements of articulation, and with others it becomes actual audible language.

§ 2. Scientific Language.

Clearly, then, accurate thinking requires the existence of accurate language. For all three purposes mentioned—for communication, for record, and for assistance in the act of thought itself, it will be best:

(1) That each word should stand for only one group of attributes.

(2) That each important group of attributes, whether regarded as an independent "thing," or as an abstraction, should have a name to itself.

Language, then, should not be ambiguous; and it should be very copious.

These conditions are not completely fulfilled in any language actually spoken by men.[1] Language has grown up in a casual way; it has not been made to serve the purposes of theorists, it is a practical instrument used by plain and ignorant people in their daily concerns. Thus it is always ambiguous. Even when a new word is borrowed from science, with an exact connotation, the journalist and the man in the street set to work to blur it, for exactness in thought is their last interest. Again, the burden of a large vocabulary is one against which the plain man rebels. Stock adjectives, like "nice" and "awful," are the result of this revolt. They are always usable, because they have next to no meaning. Most of us want to get on with as few words as possible; and if a new slang expres-

[1] See above, Chap. II. § 3.

sion or current catchword comes into use, it is at the expense of another which goes out of use. A vocabulary of a few hundred words will express all that most men, and nearly all women, want to say, and a great deal more than they want to think.

To obviate these defects in ordinary language, precise thinkers have had to invent a language of their own. The "jargon of the philosophers," which disgusted the Humanists of the sixteenth and seventeenth centuries, was a necessary condition of exact thought. "The learned gibberish" of the Scholastics, which Locke thought a resource of priestcraft, was a necessity of the case. No science has ever made any progress without a special language of its own.

Indeed Condillac, the French philosopher (d. 1780), who popularized and extended the sceptical results of Locke's "Essay," called science "une langue bien faite," a well-made language.[1] Algebra shows the utility of a well-constructed language, chiefly as a mechanical aid to thought; Zoology shows its utility chiefly as a means of record. All the sciences exemplify it, however, in some degree. And until a science has been able to develop a proper "jargon" of its own it seldom makes much progress. We have already seen how Psychology and Political Economy suffer from this defect (Chap. II. § 3). The same difficulty meets the student of Politics. The terms *law*, *sovereign*, *rights*, *freedom*, he has to take from popular language, and he finds them full of ambigui-

[1] "Langue des Calculs," p. 7.

ties. In the newer sciences there is less danger. In Organic Chemistry fresh names are constantly being introduced, and their connotation is rendered precise from the beginning. Words like *Triphenylguanidine*, *Acetaldehyde*, *Glycocoll*, may not be pretty, but they at least have the merit of definiteness, and of suggesting their meaning to experts.

§ 3. Terminology and Nomenclature.

A Terminology is a system of terms. It will include names of things and names of attributes. All the elements of which the compound wholes recognized by a science as its subject matter are composed must have a name; all the instruments and methods employed, all the qualitative phenomena observed, and all the quantitative facts also. The terminology, then, will include all words used in the description of the phenomena dealt with in the science. It must contain "a name for every important result of abstraction—that is, a name for every important common property or aggregate of common properties, which we detect by comparison of the facts."

The Nomenclature is a particular part of what is usually called the Terminology of a science. Calling what we have spoken of above the *descriptive terminology* of the science, the *nomenclature* will include the rest of the Terminology, that is, all the other specific technical words used, viz., the names of all the classes into which the science divides its subject

matter. In Zoology the nomenclature will include all subdivisions of Animal; all the names of sub-kingdoms, divisions, classes, orders, genera, species, and varieties.

It is impossible, however, to draw a clear line between the two things. The descriptive terminology and the nomenclature pass into each other. In the group of sciences we call Biology morphological terms (for instance, *muscle, bone, limb*) belong to the nomenclature of Anatomy, but to the descriptive terminology of Zoology. We cannot always draw a clear distinction between "things" and groups of attributes which we do not recognize as things.

CHAPTER XXII.

FALLACIES.

§ 1. Old Classification of Fallacies.

FALLACIES are errors of inference. Any error in inference is a fallacy, whether due to a mere blunder or to some moral source, such as bias and indifference. But while the mistake in inference is a fallacy, the origin of the mistake is not taken into account. The logician leaves to the psychologist the task of pointing out the causes which are likely to lead to the adoption of false beliefs, by predisposing to accept insufficient evidence, or to neglect certain classes of evidence.

According to Kant, a *sophism* is a fallacy designedly perpetrated in order to arrive at a conclusion known not to be true, or known to be absurd. A *paralogism* is an involuntary fallacy which deceives ourselves. But *sophism*, *paralogism*, and *fallacy* are often used, without any distinction, to mean the same thing.

The classification of the errors of reasoning is in a very unsatisfactory state, and little has been done of value since Aristotle wrote.

In his logical treatises fallacies are divided into two groups:

(1.) Fallacies *in dictione,* or as we should say, *formal* fallacies, which can be detected even by some one who has no knowledge of the subject matter of the reasoning.

(2.) Fallacies *extra dictionem,* or *material* fallacies, which can only be detected by some one having a knowledge of the special subject matter.

Formal fallacies have been divided by Whately into two sub-classes:

(a) *Purely logical fallacies,* such as arise from a breach of the rules of the syllogism.

(b) *Semi-logical fallacies* which arise from want of precision in the use of language.

This classification is unsatisfactory, since it overlooks to a great extent the errors likely to arise in the processes of Induction, and those belonging to the subsidiary processes of Definition and Division. And, besides this, the classes overlap; since some of the Material fallacies might be equally well considered as fallacies of the Semi-logical type; while what may be regarded as one of the purely logical fallacies (breach of the rule of the Syllogism which forbids an ambiguous middle term) may be regarded equally well as a Semi-logical fallacy (Equivocation).

In fact, it is generally open to us to represent a given error as due either to the preliminary processes which give us our proposition, or to the formal process of inference.

§ 2. Mill's Classification of Fallacies.

Mill has given us a more complete scheme of classification.

He first recognizes a class of fallacies of *Simple Inspection;* which includes "not only all cases in which a proposition is believed and held for true, literally without any extrinsic evidence, either of specific experience or general reasoning; but those more frequent cases in which simple inspection creates a *presumption* in favour of a proposition; not sufficient for belief, but sufficient to cause the strict principle of a regular induction to be dispensed with, and creating a predisposition to believe it on evidence which would be seen to be insufficient if no such presumption existed" ("Logic," V. ii. § 2). In this class he places many philosophic opinions which he himself rejects, as well as certain "natural prejudices," as he calls them. Obviously such a metaphysical dust-bin is a convenience for a philosopher who wants to get rid of doctrines which he thinks it waste of time to examine in detail. Amongst the venerable propositions which Mill relegates to his rubbish-heap is "that whatever is inconceivable must be false," which many great thinkers, from Descartes and Leibnitz to Professor Sidgwick and Mr. Herbert Spencer, have considered the touchstone of all knowledge and the foundation of all philosophy.

Apparently any sufficiently general and abstract proposition which Mill considered absurd might be

regarded as a fallacy of simple inspection, even although it had approved itself to minds of the greatest subtlety and power.

Passing over this point, it is obviously the case that "simple inspection creates a presumption in favour" of many propositions which are perfectly true, and which may be rigidly inferrible from other true propositions. When we hear that Lord Rayleigh and Professor Ramsay have discovered a new element, we feel a "predisposition to believe it" on evidence much slighter than we should require to make us accept the peculiar doctrines of Mr. Sinnett and Madam Blâvatsky. How far this presumption is a reasonable one is a matter for argument; but that it does exist, and that it causes us to demand a less rigid demonstration than we should require of the reality of one of Madam Blâvatsky's miracles, is not sufficient to make us consider the belief in the existence of argon, as due to a fallacy of Simple Inspection.

All other fallacies Mill calls fallacies of *Inference*, and he divides them in a way which will be easily understood from the following table, without much further explanation.

Fallacies of Inference.

- From evidence distinctly conceived . . .
  - Inductive fallacies .
    - 1. Fallacies of Observation.
    - 2. Fallacies of Generalization.
  - Deductive fallacies .
    - 3. Fallacies of Ratiocination.
- From evidence indistinctly conceived . . . . .
  - 4. Fallacies of Confusion.

In the last class he places those cases of error which arise when the premises "have never been conceived in so distinct a manner as to produce any clear consciousness by what means they were carried out; as in the case of what is called reasoning in a circle." He brings under it all the "semi-logical fallacies," and the material fallacies of *petitio principii* and *ignoratio elenchi.*

As we have already said, no classification of fallacies that has yet been proposed can be considered satisfactory. The plan we shall adopt is to give the traditional list of fallacies, the names of which have become classical, and have become part and parcel of ordinary and non-technical vocabulary.

§ 3. Purely Logical Fallacies.

(1) The chief of these are infractions of the rules of the Syllogism, *e.g.* (*a*) Fallacy of four terms (*quaternio terminorum*) which is a breach of Rule 1; but usually

arises from the use of an ambiguous middle term, which is forbidden by Rule 3. (*b*) Fallacy of undistributed middle. (*c*) Fallacyof illicit process. (*d*) Fallacy of negative premises, and those due to a breach of Rule 6.

(2) But to these syllogistic fallacies, we must add the *fallacies of Immediate Inference,* such as simple conversion of universal affirmatives, arising from illicit distribution of an undistributed term.

(3) *Fallacies of Conditional reasonings,* such as *denying the antecedent,* and *affirming the consequent.*

(4) Fallacies of Definition, such as the fallacy known as *circulus in definiendo.*

(5) Fallacies of Division, such as *cross-division.*

§ 4. Semi-Logical Fallacies.

These are all "fallacies of confusion," to use Mill's term; due to the use of language wanting in precision.

The old Logics mention six, but they are of very various importance.

(1.) *Fallacy of Equivocation,* in which the same word is used, but is used in two or more different senses. As we have seen, all, or nearly all, words are equivocal in some degree, and our first care must be to see that the words that we use have exactly the same meaning in all the propositions of a reasoning. The most frequent case is that of ambiguous middle term;[1] but we may have a term used in a different sense in the conclusion and in the premise in which it occurs.

[1] See p. 252, above.

(2.) *Fallacy of Amphibology,* in which a sentence is so constructed as to be ambiguous from its form. The Latin construction of accusative and infinitive is specially liable to this disadvantage. In English the careless use of pronouns, and specially of relative pronouns, is a frequent cause of ambiguity. The following verbal tangle is from a daily newspaper: "Her own story was that she had a quarrel with the deceased, first about her wages, and secondly about the soup, and that she seized the deceased by the throat, and she fell, and when she got up she was looking for something to strike her with, and upon this she struck the deceased a blow in the throat, and she fell and died almost instantaneously."

As English is scarcely an inflected language, the collocation of words is of great importance. A schoolgirl once wrote in an examination paper, "The 'Iliad' is the story of the siege of Troy by Homer;" but it was a popular novelist who told us that a gentleman "drove away from the church where he had been married in a coach and six."

(3.) *The Fallacy of Composition* is that in which ambiguity arises from taking a middle term distributively in one proposition and collectively in another, without due notice. Thus, we may have a middle term used distributively in the major, and collectively in the minor premise;[1] or a term may be used distributively in a premise, and collectively in the conclusion.

[1] See p. 252, above.

When we argue that because a trades-union can artificially raise wages in one trade, all the trades unions can artificially raise wages in all trades by acting together, we commit this fallacy; for a trades-union can often only raise wages in its own trade at the expense of the welfare of other groups of workmen.

(4.) *Fallacy of Division.* If, on the contrary, we proceed from a collective use of a term to a distributive use of it, without due notice, we have the fallacy of division. If we argue that because Catholics hold that all the bishops of the Church collectively cannot err, therefore they must hold that any individual bishop is infallible, we attribute to them this fallacy.

(5.) *Fallacy of Accent.* Ambiguity may arise from false accent. Jevons cites the time-honoured case from 1 Kings, xiii. 27: "And he spake to his sons saying, Saddle me the ass. And they saddled *him*." Some unfortunate reader mistook the italics, which indicate that the word "him" is not in the original Hebrew, as a mark of accent.

(6.) *Fallacy of Figure of Speech*, in which some colloquial or some rhetorical expression is taken literally, or *vice versâ*.

§ 5. Material Fallacies.

Aristotle mentions seven kinds of fallacies *extra dictionem*, that is, fallacies not lying in the method of expression, but due to the subject matter.

They are as follows:

(1.) *Fallacy of Accident,* also called the fallacy *a dicto simpliciter ad dictum secundum quid.* Here the error consists in inferring, that what is true generally of a class of things, is true in any given special case; which may be marked by some individual or specific peculiarity, rendering the predicate inapplicable. It is the fallacy most likely to occur when we apply a general law to a particular case. It is the besetting sin of the political idealist and doctrinaire. The man often commits it who solves all problems by the application of some wide formula, such as "Peace is the greatest of all blessings," or, "Liberty is the most valuable possession a man can have," or, "Order is the one thing politically needful." So do those who argue (as John Bright and the *laisser-faire* Liberals did) that because interference with free contract is undesirable, some particular factory legislation is undesirable, without looking at the special circumstances of the case.

(2.) *Converse fallacy of Accident,* or the fallacy which proceeds *a dicto secundum quid ad dictum simpliciter.* When a statement has been made with an explicit, or implicit, condition, we must not, of course, repeat it without the condition. In so doing, we generalize from a special case, which may afford no ground whatever for the inference. Just as in the fallacy of accident we apply a general law without caution to a case which does not really come under it, we here infer a general law from a case which is not really an example of the law.

When the Puritan settlers in New England passed their three famous resolutions—"Resolved, *first*, that the Earth is the Lord's and the fulness thereof; *secondly*, that He hath given it to His Saints; *thirdly*, that we are His Saints"—they committed the fallacy at least twice. Whenever a text is torn from its context and is stated as a general truth without reference to the particular circumstance under which it appears in Scripture the fallacy is committed.

Both these fallacies of Accident then are due to the confusion between the general and the special. We forget that almost all statements are implicitly conditional, that they presuppose a number of conditions which they do not assert.

If a physician tells us that a particular drug will cure a certain complaint, he tacitly implies a large group of favourable circumstances. It will not cure the complaint under all circumstances whatever. It is only the ignorant and foolish man, who, finding the medicine serviceable in his own case, will attempt to persuade everybody else who suffers from the same disease to take it. Hence the need of constantly reminding us that "S is P" only *mutatis mutandis*, or only *cæteris paribus*.

On the other hand, absolute precision of language, even if attainable, would be dearly purchased at the cost of intolerable verbosity. We must make rough general propositions, and we must apply them. But all the same we must remember, in the words of the homely proverb, that "circumstances alter cases."

Lotze rightly points out that both fallacies of accident can be brought under the fallacy of ambiguous middle. The reader will have no difficulty in testing this statement.

(3.) *Ignoratio Elenchi.* The *elenchus* was the technical term for a syllogism put forward with a view to confute an adversary. If the adversary declined to notice the *elenchus,* and continued to argue at large without any reference to our refutation of his thesis, he was said to commit the fallacy of ignoring the *elenchus.* The meaning of *ignoratio elenchi* has been widened to cover all cases of answering to the wrong point, of refusing to notice the argument of an opponent, and continuing to prove some other point than that which he has attacked. Whately called this wider form, the *fallacy of Irrelevant Conclusion.*

All such rhetorical shifts as the *argumentum ad hominem*—in which, instead of disproving that "S is P," we show that our opponent is not the right man to bring forward the statement "S is P," or that he is dishonest, or that he is incapable—come under it. So does the *argumentum ad verecundiam,* the appeal to the veneration in which some person who holds the doctrine we support is rightly held, and the insignificance of the objector.

The *argumentum ad populum* may be regarded as another type of the *ignoratio elenchi,* in which the real point in dispute is kept out of sight by an appeal to the special prejudices of the ignorant and the foolish, who form the majority of every large body of men.

§ 6. Petitio Principii and other Material Fallacies.

There are four other fallacies *extra dictionem* to be mentioned.

(4.) *Petitio Principii*, or *Begging the Question*, means the employment, for the purpose of proof, of a premise which itself is only true on the assumption that the conclusion is true. It is sometimes called *circulus in probando.*

This fallacy often takes the form of using what Bentham called a "question-begging epithet;" some word—which implies or suggests the attribute we wish to prove of the subject—is employed to describe the subject. *Heretical*, *universal*, *unconstitutional*, are favourite question-begging appellatives. A moralist who seeks to show that we may never tell an untruth in the interests of euphemism because all telling of untruths is *lying*, really assumes the whole point at issue; for a *lie* does not merely mean untruth, but immoral and indefensible untruth.

As Whately remarks:—"The English language is perhaps the more suitable for the fallacy of *petitio principii*, from its being formed from two distinct languages, and thus abounding in synonymous expressions, which have no resemblance in sound, and no connection in etymology;" so that a sophist may bring forward a proposition expressed in words of Saxon origin, and give as a reason for it, the very same proposition stated in words of Latin origin; *e.g.*, "to

allow every man an unbounded freedom of speech must always be, on the whole, advantageous to the State; for it is highly conducive to the interests of the community, that each individual should enjoy a liberty, perfectly unlimited, of expressing his sentiments."

In the same way, scientific men are exposed to the danger of fancying that a statement in highly technical terms, derived from Greek or Latin, of the fact to be explained, constitutes an explanation of it. We cannot legitimately explain why children are sometimes subject to the same diseases as their parents by the use of such terms as *heredity*. The difficulty lies in the fact that some diseases appear to be transmitted, while others apparently are not. We do not give a reason for the former by saying in technical language that they *are* transmitted.

(5.) *Non Causa pro Causa*. This is the fallacy of false generalization. In some cases, a *sign* or *symptom*, C, from which the presence of B may legitimately be inferred, is taken as the cause of B, when it is really a concomitant part of the effect of a cause A. Thus some of the symptoms of a disease are often regarded as the cause of the rest. As Oesterlen says:—"We are in danger of confounding cause and effect together if we assert that grief may give rise to disease of the stomach, and even to cancer of that organ; or that the habit of sleeping after dinner causes obesity, congestion of the brain, and apoplexy."[1]

Even if a fact, A, is known to precede another fact,

[1] "Medical Logic," Sect. VII.

B, sometimes, or even always, we cannot necessarily infer that A is the cause of B. Against this fallacy of false generalization, known as that of *post hoc ergo propter hoc*, all the apparatus of inductive methods, the cautions as to the use of hypotheses, and the rigid rules as to the need of verification, are directed.

(6.) *Fallacy of non sequitur.* Under this might be brought all fallacies of inference, that is, all fallacies whatever, and no advantage is gained by regarding it as a separate class.

(7.) *Fallacy of Many Questions.* Aristotle points out that questions are often asked to which a single "Yes" or "No" cannot be answered. Before such questions can be answered they must be analyzed into their constituent parts.

Barristers practising before weak or lazy judges sometimes ask unfair questions of this sort, with the object of getting some unpleasant admission from a witness. "Have you left off beating your mother?" is a crude specimen of this form of forensic wit. But whenever we try to pin a man down to the use of some ambiguous epithet, with the object of turning the admission against him, we commit the same fallacy, or rather sophism. Thus, if we ask a High Church clergyman if he is a Protestant, or demand from a Broad Churchman if he believes in the inspiration of the Bible, we ask questions which can only be properly answered after some preliminary explanation, and not with a simple affirmative or negative. The old scholastic writers used to say in such cases, "Dis-

tinguo," "I make a distinction;" and this is the best way of dealing with the matter.

§ 7. Fallacies of Observation.

In his first class, *fallacies of Observation,* Mill recognizes two subdivisions—fallacies of *non-observation* and fallacies of *mal-observation.* In the former we overlook what is before us to be observed; our error is of a negative character. In the latter we perceive wrongly, and make a positive mistake by apparently perceiving what is not there to be perceived.

We have seen that in all perception there is implicit and sub-conscious inference (Chap. I. § 1). But it is doubtful whether we can lay down any principles underlying valid observation. Not only is the process sub-conscious, but it eludes retrospective analysis; we cannot reinstate in clear consciousness the steps by which the result was obtained. The process of reasoning is also sometimes practically unconscious, but in reasoning we can usually bring clearly before the mind the phases or elements of the complex process in such a way that we can test their validity. In observation there is no means of doing this. We cannot even draw the line between what is directly given us in perception, as it were, from *outside,* and what we add to it. To a few hints in the shape of visual sensation we join an enormous mass of sub-conscious inference. But we cannot draw the line where the one begins and the other leaves off.

Although we can give general rules for avoiding error in observation, we cannot formulate any principles which must be complied with if observation is to be correct. There is no logic of observation.

Indeed, Logic begins where observation leaves off. It deals with such knowledge as has been expressed in propositions.

APPENDIX A.

BOOKS RECOMMENDED.

I. GENERAL.

Welton: *Manual of Logic.* 2 vols.
Bain: *Logic.* 2 vols.
Fowler: *Logic.*
Mill: *System of Logic.*

II. SPECIAL.

Jevons: *Principles of Science.*
Keynes: *Formal Logic.*
Venn: *Symbolic Logic.*
,, *Logic of Chance.*
,, *Empirical Logic.*
De Morgan: *Formal Logic.*
Boole: *Investigation of the Laws of Thought.*
Mansel: *Prolegomena Logica.*
Whewell: *Novum Organum Renovatum.*

III. FOR FURTHER READING (PRINCIPALLY CRITICAL).

Bradley, H.: *Principles of Logic.*
Bosanquet: *Logic or the Morphology of Knowledge.*

Lotze: *Logic.* 2 vols. (transl.).
Sigwart: *Logic.* 2 vols. (transl.).
Hamilton: *Lectures on Logic.* 2 vols.
Mansel: Introduction and Appendix to his edition of Aldrich's *Artis Logicæ Rudimenta.*

IV. MAINLY OF HISTORICAL INTEREST.

Aristotle: *Organon* (various logical treatises, of which there is a translation in Bohn's Library).
Bacon: *Novum Organum* (ed. by Fowler).
Arnauld: *L'Art de Penser* (translated by T. S. Baynes as the *Port-Royal Logic*).
Whewell: *On the Philosophy of Discovery.*
" *History of the Inductive Sciences.*

APPENDIX B.

EXAMPLES FOR SOLUTION.

I. IMMEDIATE INFERENCE.

Put each of the following into exact logical form, and give its obverse, converse (where possible), and contrapositive (where possible).

1. All wise men are modest.
2. The Jews are monotheists.
3. No mortals are perfectly happy.
4. Some lawyers are not honest.
5. All lawyers are not knaves.

6. All 's well that ends well.
7. Whatever is, is right.
8. No smoking allowed.
9. All living tissue is organic.
10. Some murmur when their sky is clear.
11. A fool at forty is a fool indeed.
12. All cannot receive this saying.
13. All the perfumes of Arabia will not sweeten this little hand.
14. Some acids do not contain oxygen.
15. They laugh that win.
16. The square of four is sixteen.
17. To be good is to be happy.
18. Brutus killed Cæsar.
19. No end of people were there.
20. All hope abandon, ye who enter here.
21. The better the day the better the deed.
22. No small dissension arose.
23. All the witnesses were not trustworthy.
24. None knows where the shoe pinches but the wearer.
25. No man is a hero to his valet.
26. Milton wrote "Paradise Lost."
27. Only the actions of the just
    Smell sweet and blossom in the dust.
28. Knowledge is power.
29. Some are born great.
30. Who drives fat oxen should himself be fat.
31. A bank-note is an order to pay gold on demand.
32. No men are braver than some blacks.
33. All exercises are not so easy as they look.
34. He who is capable of making a pun is capable of picking a pocket.

35. Μέγα βιβλίον μέγα κακόν.
36. Reading maketh a full man.
37. Life is not all beer and skittles.
38. Some of the guests were not all they should be.
39. Beati possidentes.
40. He is gentle that doth gentle deeds.
41. What 's yours is mine, what 's mine is my own.
42. There is no mind without motion.
43. All I want is justice.
44. Most women have no character at all.
45. There are criminals who deserve reward.

II. MEDIATE INFERENCE AND FALLACIES.

Put the following reasonings into exact logical form, identify the form, and where any fallacy (or apparent fallacy) occurs, explain it:

1. My letter has not been answered, therefore it must have miscarried.
2. He cannot be a gentleman, for no gentleman would do such a thing.
3. The whale is warm-blooded, and is therefore not a fish.
4. It is going to be hot, for there is a haze on the hills.
5. These books are all by X, therefore most of them are sure to be rubbish.
6. The members of the club are not all good players, for Jones belongs to it.
7. He must be musical, for he is always going to concerts.

8. Things-in-themselves are of no importance, seeing that they are not knowable.
9. A salaried clerk may be at the same time a man of letters: for Charles Lamb was both.
10. Every criminal is more or less insane: but the insane ought not to be punished: therefore no criminal should be punished.
11. The cruel are not always cowardly: for Richard III., Peter the Great, etc., were brave.
12. The company must certainly be a sound one, because the Chairman is a peer.
13. She is a woman, therefore may be woo'd;
She is a woman, therefore may be won.
14. Many educated men do not write good English, for not even all University men do so.
15. Members of Parliament are not chosen for independence of judgment, though such men are best qualified for the office: therefore few of those best qualified become members.
16. The books in the library are all novels, which are not what I care for: consequently no books that I care for *are* in it.
17. Among those who distinguished themselves were some recruits, since all the troops (many of whom were recruits) did so.
18. All these men have passed, which shows that some very stupid persons manage to do so.
19. Some eminent men have not obtained university distinction, which is therefore no conclusive proof of real ability.
20. No really musical person would applaud that, though some of the audience are doing so.

21. Some of the books had evidently not been read, for they had not even been cut.
22. As some of the witnesses contradicted each other, there must have been perjury somewhere.
23. The whole family has been vaccinated, yet some have had small-pox; therefore vaccination is no safeguard.
24. None of the family have been vaccinated and none have had small-pox; therefore vaccination is unnecessary.
25. Many Englishmen have foreign surnames, and must therefore have been originally of foreign extraction.
26. Some of these books are not well bound, for they are going to pieces as no well-bound books would do.
27. Some of these books will not be much read—they are too abstruse.
28. This is something I don't know, and is therefore not knowledge.
29. Teetotallers do not use alcoholic drinks, and this man does not do so: therefore he is a teetotaller.
30. He that is of God heareth God's words; ye therefore hear them not, because ye are not of God.
31. Practice makes perfect: therefore a neighbour who practises all the time is a perfect neighbour.
32. To play all day is a proof of great idleness: so this violinist must be a very idle person.
33. That no reasonable man holds this view is not disproved by the fact that Mr. X. holds it.
34. None of the party suffered, though some drank the water of the place: therefore this does not always cause mischief.
35. No English peer can sit in the House of Commons: for he is *ipso facto* a member of the House of

Lords, and no man can be a member of both Houses.

36. If I write long letters he is bored: if short, he is offended: therefore I won't write at all.
37. He cannot have been there—otherwise I should have seen him.
38. A London graduate is sure of the post: I am a London graduate, and am therefore sure of it.
39. Only contented people are wise: therefore the tramp contented in his rags is a wise man.
40. You are not what I am: I am a man: therefore, you are not a man.
41. One patent stove saves half the ordinary amount of fuel: therefore two would save all.
42. If virtue is voluntary, vice is voluntary; virtue is voluntary, therefore so is vice.
43. A B and C D are each of them equal to E F: therefore they are equal to each other.
44. He is the greatest lover of any one who seeks that person's greatest good; a virtuous man seeks the greatest good for himself; therefore a virtuous man is the greatest lover of himself.
45. Warmth is agreeable, therefore cold is disagreeable.
46. Warm countries alone produce wines: Spain is a warm country; therefore Spain produces wines.
47. All that glitters is not gold: tinsel glitters, therefore it is not gold.
48. Every hen comes from an egg; every egg comes from a hen; therefore every egg comes from an egg.
49. What we eat grew in the fields; loaves of bread are what we eat; therefore loaves of bread grew in the fields.

50. He who is most hungry eats most; he who eats least is most hungry; therefore he who eats least eats most.

51. Wine is a stimulant; therefore, in a case where stimulants are hurtful, wine is hurtful.

52. Theft is a crime; theft was encouraged by the laws of Sparta; therefore the laws of Sparta encouraged crime.

53. Either you or I must be mistaken:
   (1) I am not: therefore you must be.
   (2) You are: therefore I am not.

54. If he is drunk, he is incapable.
   (1) He is drunk, and therefore is incapable:
   (2) He is not drunk, and therefore is not incapable:
   (3) He is incapable, and therefore is drunk:
   (4) He is not incapable, and therefore is not drunk.

55. No one is a better cricketer than W. G.: W. G. is a better cricketer than you: therefore no one is a better cricketer than you.

56. All Laplanders are poets; Homer was a Laplander; therefore Homer was a poet.

57. All these men are quite sufficient for the job; you are one of them, and are therefore quite sufficient for it.

58. No man can serve two masters; for either he will hate the one and love the other; or else he will hold to the one, and despise the other.

59. What is done can never be undone; therefore this knot can never be undone.

60. There is a pressure of one pound on any square inch of the interior surface of this vessel; therefore there is a pressure of one pound on every square inch of the surface.

61. Since the end of poetry is pleasure, that cannot be unpoetical with which all are pleased.
62. The Divine Law commands us to honour kings; Louis XIV. is a king; therefore the Divine Law commands us to honour Louis XIV.
63. None but Whigs vote for Mr. B. All who vote for Mr. B. are ten-pound householders. Therefore none but Whigs are ten-pound householders.
64. The members of the board were all either bondholders or shareholders, but not both; and the bondholders, as it happened, were all on the board.
65. Given that everything is either Q or R, and that all R is Q, unless it is not P, prove that all P is Q.
66. Given that P is Q R, and that p is q r; show that Q is P R, (and) R is P Q.
67. It is known of certain things that (1) where the quality A is, B is not; (2) where B is, and only where B is, C and D are. Derive from these conditions a description of the class of things in which A is not present, but C is.

APPENDIX C.

LONDON B.A. QUESTIONS.

1. Distinguish hypothesis from theory. Explain the use of hypothesis in scientific procedure. Show, by a concrete example, how far the imagination, and how far the reason, has entered into the construction of a

workable hypothesis. By what criticism would you test all hypotheses? (B.A. 1890.)

2. Give a detailed definition of the following terms from your own point of view: thought, feeling, sensation, perception, judgment, reasoning, proof, verification, evidence. (B.A. 1890.)

3. Examine the following: "Every bird comes from an egg; every egg comes from a bird; therefore every egg comes from an egg." (B.A. 1890.)

"No reason without language."

"No language without reason."

Comment critically on these aphorisms. (B.A. 1890.)

4. Distinguish analysis from synthesis in full detail. Explain the function and value of each in the scientific study of phenomena, and the way in which they co-operate in the formation of a scientific theory of any group of phenomena. (B.A. 1890.)

5. Distinguish the various kinds of logical inference, and show their relations to one another. (B.A. 1890.)

6. Examine:

(α) "All responsible beings are rational; responsibility increases with the increase of rationality; some dogs are more rational than some men; therefore some dogs are more responsible than some men."

(β) "If I am to pass this examination, I shall pass it, whether I answer correctly or not: if I am not to pass it, I shall fail, whether I answer correctly or not: therefore, it is of no consequence how I answer the questions." (B.A. 1890.)

7. Distinguish between the psychological and the logical treatment of thought, with special reference to the

question whether all our thinking is carried out by concepts, as the logician understands them. (B.A., 1891.)

8. Give the contradictory, also the converse, the obverse and the contrapositive of each of the following propositions:

   (*a*) It cannot be doubted that every man pursues his own interest.

   (*b*) The writer of the document was A. B.

   (*c*) About one in three of the candidates were University mèn. (B.A., 1891.)

9. Give a brief account of the predicables, and point out any difficulties in adjusting them to modern scientific thought. (B.A., 1891.)

10. Is Logic bound by the ordinary usages of language? Discuss the question in connection with the following topics:

    (*a*) The quantification of the predicate.

    (*b*) The proper logical interpretation of the form, "Some S is P." (B.A., 1891.)

11. What do you consider to be the real distinction between a categorical and a hypothetical proposition? Are the processes of immediate inference applicable to hypothetical propositions? (B.A., 1891.)

12. Discuss, both logically and historically, the relation of induction to deduction. (B.A., 1891.)

13. Bring out the logical peculiarities of mathematical reasoning, and inquire whether it is radically distinct from other forms of scientific reasoning. (B.A., 1891.)

14. Unfold the nature of hypothesis, assigning (if you can) a definite meaning to the expressions "legitimate

hypothesis" and "vera causa." Is hypothesis an essential factor in inductive investigation? (B.A., 1891.)

15. What do you understand by Laws of Thought? What different views have been held as to their origin and significance? How would you distinguish them from Laws of Nature? (B.A., 1892.)
16. Distinguish verbal from real Predication, and show how the five predicables bring out the distinction. (B.A., 1892.)
17. State and compare some of the most important methods of classifying Fallacies known to you. (B.A., 1892.)
18. In what relation does the causal judgment stand to the Uniformity of Nature? (B.A., 1892.)
19. How many distinct methods of experimental research does Logic recognize? Give examples of each and show how they co-operate. (B.A., 1892.)
20. Illustrate the logical principles of classification from any one of the sciences of Chemistry, Botany, or Physics. (B.A., 1892.)
21. Amplify, and apply a logical test to, the following arguments:
    (*a*) The theory of evolution is true, because it is accepted by every scientific biologist.
    (*b*) A good temper is a sign either of a good conscience or of a good digestion; therefore the conscientious and the healthy will always possess a good temper.
    (*c*) To call a person an animal is to speak the truth; therefore to call him an ass (which is to call him an animal) is to speak the truth.
    (*d*) The laws of Nature can never be broken. Social

law is a part of the general system of Nature; therefore it cannot be broken. (B.A., 1892.)

22. Give the contradictory, the obverse, the converse, and the contrapositive of the following:
(*a*) Private vices are public benefits.
(*b*) Not to know me argues thyself unknown.
(*c*) Beauty and use are identical.
(*d*) No man is always consistent. (B.A., 1892.)

23. Bring out the exact scope of Logic, defining its relation (1) to the special sciences; (2) to psychology; (3) to philosophy or theory of knowledge. (B.A., 1893.)

24. What is meant by saying that Logic deals only with the *form* of thought? Show how the use of symbols enables us to examine the form of our thought. (B.A., 1893.)

25. Bring out fully the peculiarities of the class-symbol *not-A*. What is meant by calling it infinite? Is it found in the propositions of every-day life? (B.A., 1893.)

26. State and illustrate what you understand by Obversion. On what laws or axioms does the validity of this process depend? (B.A., 1893.)

27. Explain fully the limitations of the conclusions obtainable in the Third Figure of the Syllogism. Are these limitations got rid of by applying Obversion (Permutation) to the premises? (B.A., 1893.)

28. Discuss the proposition that the cause invariably precedes the effect. Have recent discussions served to confirm Mill's view on this subject? (B.A., 1893.)

29. What is a Coincidence, and how can it be distinguished

from a real connection of events? What would you understand by the "explanation of a coincidence"? (B.A., 1893.)

30. Discuss the requirements of Definition as applied to scientific terms. How far do the rules of formal definition carry us in this case? (B.A., 1893.)
31. Bring out the meaning, and estimate the logical value, of the three Laws of Thought. (B.A., 1894.)
32. State the different ways in which Terms may be classified, giving an illustration of each. Have all the distinctions equal logical importance? (B.A., 1894.)
33. On what grounds has the Quantification of the Predicate been maintained? Estimate these grounds critically. (B.A., 1894.)
34. State, and critically examine, the various ways in which Induction has been said to differ from Deduction. (B.A., 1894.)
35. Explain the following fallacies, giving an example of each: ignoratio elenchi, non causa pro causa, a dicto secundum quid, ἀμφιβολία, false analogy, malobservation. (B.A., 1894.)
36. Briefly explain, and give an account of, sorites, epicheirema, inductio per simplicem enumerationem, nota notæ, a crucial instance, elimination of chance. (B.A., 1894.)
37. What do you understand by classification in Science? Show how the Sciences have been advanced, as the principles of classification have improved. (B.A., 1894.)
38. Examine the logical form and validity of the following arguments:

    (*a*) A fish is cold-blooded and breathes by gills;

neither of these things is true of a whale; therefore it is not a fish.

(*b*) A is never found without B, and B is never found without C; therefore C is never found without A.

(*c*) To assault another is wrong; consequently a soldier who assaults an enemy does wrong. (B.A., 1894.)

39. In what different ways has the relation of Logic to Psychology been conceived? Give your own view of the distinction and of the connection between them. (B.A., 1895.)

40. Discuss the meaning and the logical importance of the distinction between Concrete and Abstract Terms. Explain fully how it happens that it is sometimes difficult to say to which of these classes a particular Term should be referred. (B.A., 1895.)

41. Bring out the meaning of each of the following accounts of the proposition, "All men are mortal," and say which is logically to be preferred:

(*a*) All men have the attribute mortality.

(*b*) Men = mortal men.

(*c*) Men form part of the class mortals.

(*d*) If a subject has the attributes of a man, it has also the attribute mortality. (B.A., 1895.)

42. Give a brief account of the several kinds of Immediate Inference, pointing out which are reversible, and showing on what principles or assumptions their legitimacy depends. (B.A., 1895.)

43. Explain the syllogistic rules respecting two negative and two particular premises, pointing out the grounds on which they rest. Do the following break either of these rules?

(*a*) This person is very learned, and also very sociable; consequently some very sociable persons are very learned.

(*b*) No man is a proper object of contempt; at the same time no man is perfectly admirable; consequently some beings who are not perfectly admirable are not proper objects of contempt.

(*c*) The majority of English people have but little literary taste; and the majority of English people read; from which it follows that some who read have but little literary taste. (B.A., 1895.)

44. State what you consider the best definition of Cause, as required for logical purposes. Has the recent development of scientific conceptions affected the logical doctrine of Causation? (B.A., 1895.)

45. What does the Logician understand by Explanation? What different kinds are there? Can we be certain that any scientific explanation is complete and final? (B.A., 1895.)

46. Discuss the connection between Hypothesis and Inductive Investigation; and show, by means of an example, what constitutes the complete verification of a Hypothesis. (B.A., 1895.)

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